Principal Components (PCA) & Exploratory Factor Analysis (EFA) with SPSS

IDRE Statistical Consulting

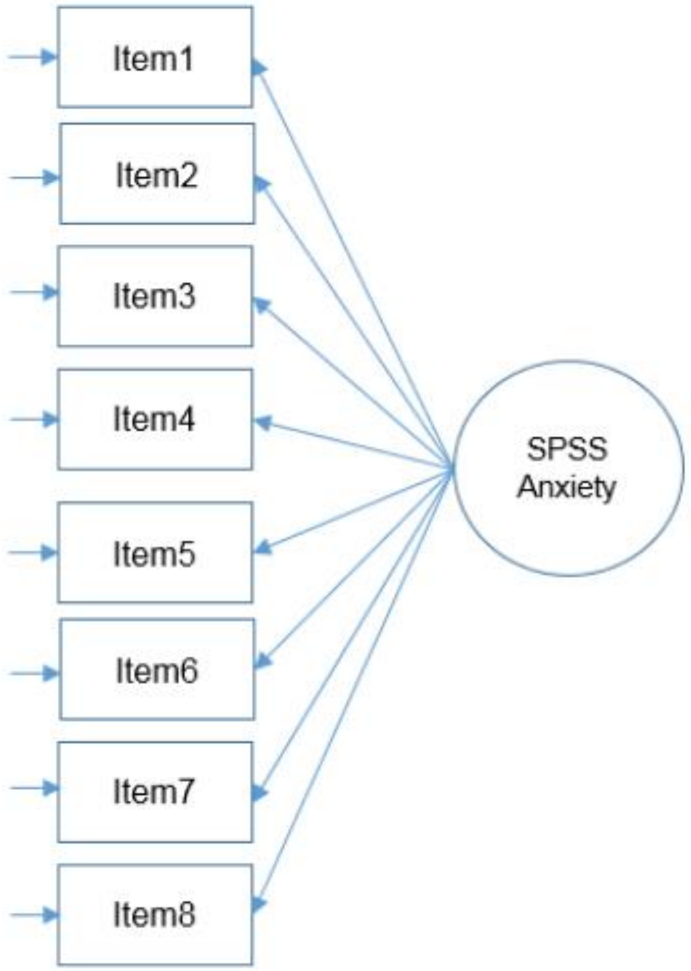
Outline

- Introduction
  - Motivating example: The SAQ
  - Pearson correlation
  - Partitioning the variance in factor analysis
- Extracting factors
  - Principal components analysis
    - Running a PCA with 8 components in SPSS
    - Running a PCA with 2 components in SPSS
  - Common factor analysis
    - Principal axis factoring (2-factor PAF)
    - Maximum likelihood (2-factor ML)
- Rotation methods
  - Simple Structure
  - Orthogonal rotation (Varimax)
  - Oblique (Direct Oblimin)
- Generating factor scores

Introduction

- Motivating example: The SAQ
- Pearson correlation
- Partitioning the variance in factor analysis

Factors and Items



SPSS Anxiety Questionnaire (SAQ-8)

1. I dream that Pearson is attacking me with correlation coefficients
2. I don’t understand statistics
3. I have little experience with computers
4. All computers hate me
5. I have never been good at mathematics
6. My friends are better at statistics than me
7. Computers are useful only for playing games
8. I did badly at mathematics at school

Pearson Correlation of the SAQ-8

There exist varying magnitudes of correlation among variables

Correlations

	Statistics makes me cry	My friends will think I'm stupid for not being able to cope with SPSS	Standard deviations excite me	I dream that Pearson is attacking me with correlation coefficients	I don't understand statistics	I have little experience of computers	All computers hate me	I have never been good at mathematics
Statistics makes me cry	1							
My friends will think I'm stupid for not being able to cope with SPSS	-.099	1						
Standard deviations excite me	-.337	.318	1					
I dream that Pearson is attacking me with correlation coefficients	.436	-.112	-.380	1				
I don't understand statistics	.402	-.119	-.310	.401	1			
I have little experience of computers	.217	-.074	-.227	.278	.257	1		
All computers hate me	.305	-.159	-.382	.409	.339	.514	1	
I have never been good at mathematics	.331	-.050	-.259	.349	.269	.223	.297	1

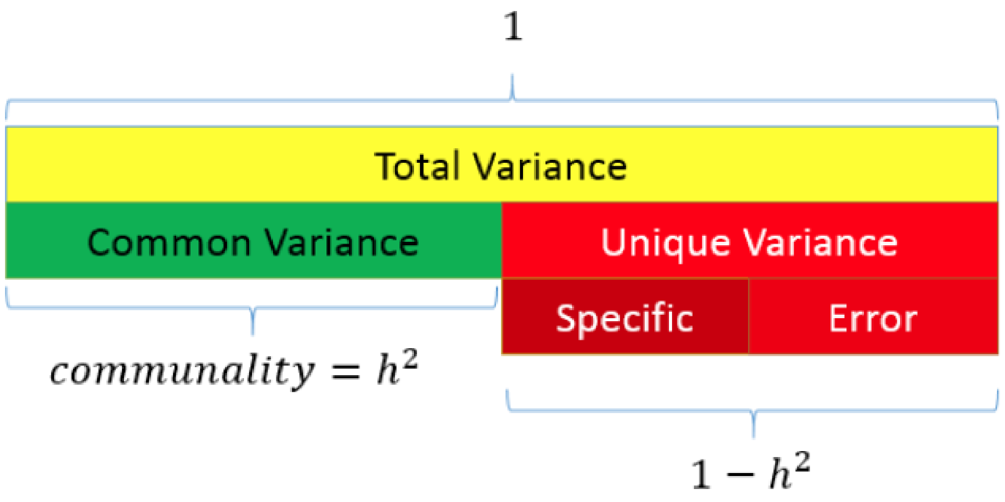
Large negative

Large positive

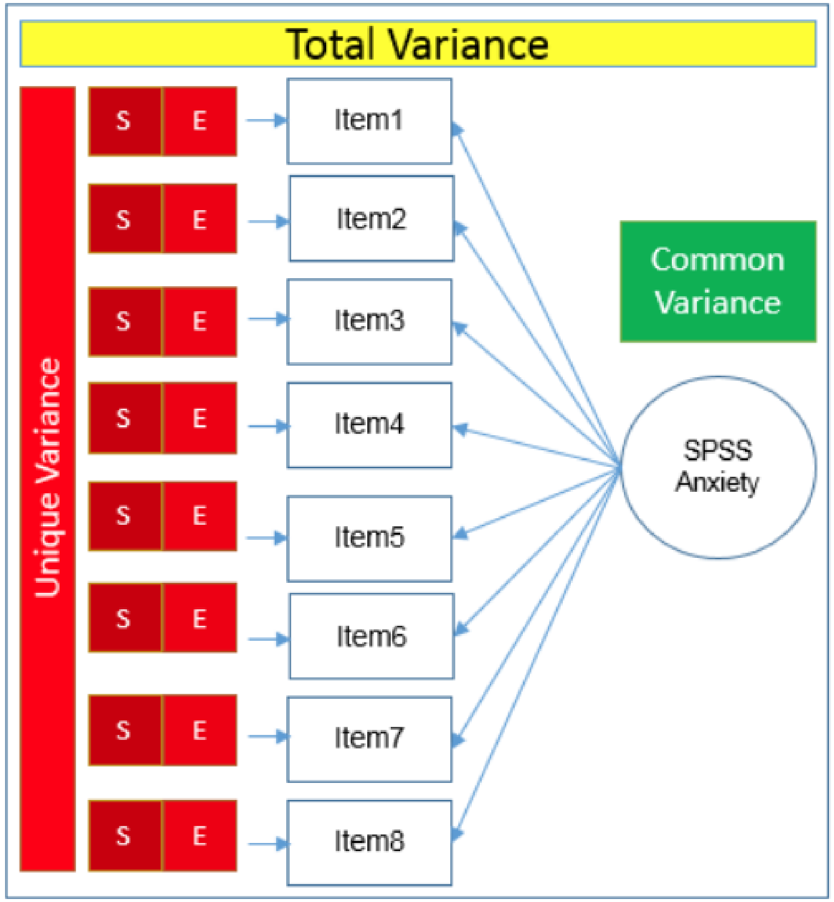
Partitioning the variance in factor analysis

- **Common variance**
  - variance that is shared among a set of *items*
  - **Communality** (h^2)
    - common variance that ranges between 0 and 1
- **Unique variance**
  - variance that's not common
  - **Specific variance**
    - variance that is specific to a particular item
    - Item 4 "All computers hate me" → anxiety about computers in addition to anxiety about SPSS
  - **Error variance**
    - anything unexplained by common or specific variance
    - e.g., a mother got a call from her babysitter that her two-year old son ate her favorite lipstick).

Variance Partitioning in an EFA



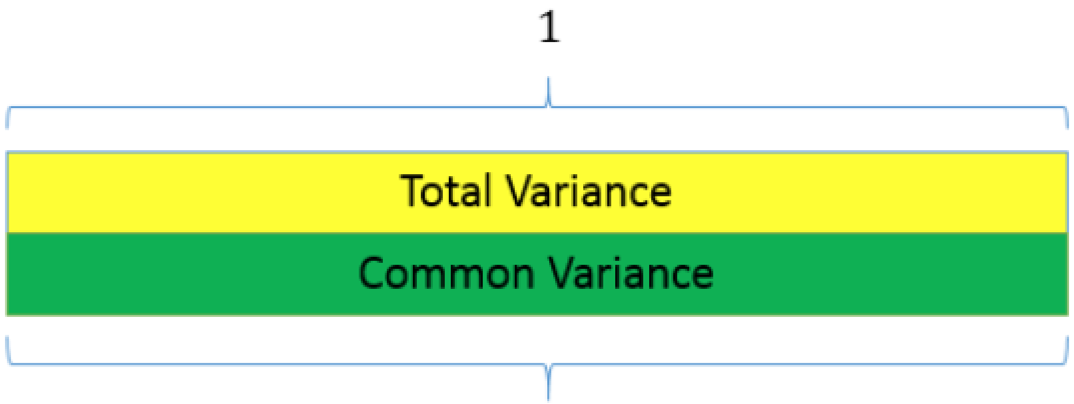
Total variance is made up of common and unique variance



Common variance = Due to factor(s)

Unique variance = Due to items

Variance Partitioning in PCA



$$communality = h^2 = 1$$

In PCA, there is no unique variance. Common variance across a set of items makes up total variance.

Extracting Factors

- Principal components analysis
  - PCA with 8 / 2 components
- Common factor analysis
  - Principal axis factoring (2-factor PAF)
  - Maximum likelihood (2-factor ML)

PCA: Eigenvalues and Eigenvectors

- **Eigenvalues**
  - Total variance explained by given principal component
  - Eigenvalues > 0, good
  - Negative eigenvalues → ill-conditioned
  - Eigenvalues close to zero → multicollinearity
  - Sum of squared component loadings across all items for each component
    - Total variance explained by principal component.
- **Eigenvectors**
  - weight for each eigenvalue
  - eigenvector times the square root of the eigenvalue → component loadings
  - **Component loadings**
    - correlation of each item with the principal component

PCA

- **Principal Components Analysis (PCA)**
  - Goal: to replicate the correlation matrix using a set of components that are fewer in number than the original set of items

Correlations

	Statistics makes me cry	My friends will think I'm stupid for not being able to cope with SPSS	Standard deviations excite me	I dream that Pearson is attacking me with correlation coefficients	I don't understand statistics	I have little experience of computers	All computers hate me	I have never been good at mathematics
Statistics makes me cry	1							
My friends will think I'm stupid for not being able to cope with SPSS	-.099	1						
Standard deviations excite me	-.337	.318	1					
I dream that Pearson is attacking me with correlation coefficients	.436	-.112	-.380	1				
I don't understand statistics	.402	-.119	-.310	.401	1			
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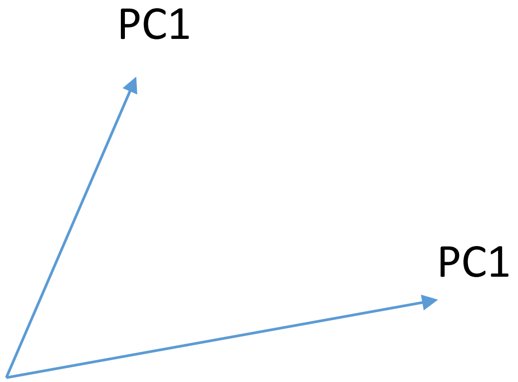
All computers hate me	.305	-.159	**-.382**	.409	.339	**.514**	1	
I have never been good at mathematics	.331	-.050	-.259	.349	.269	.223	.297	1

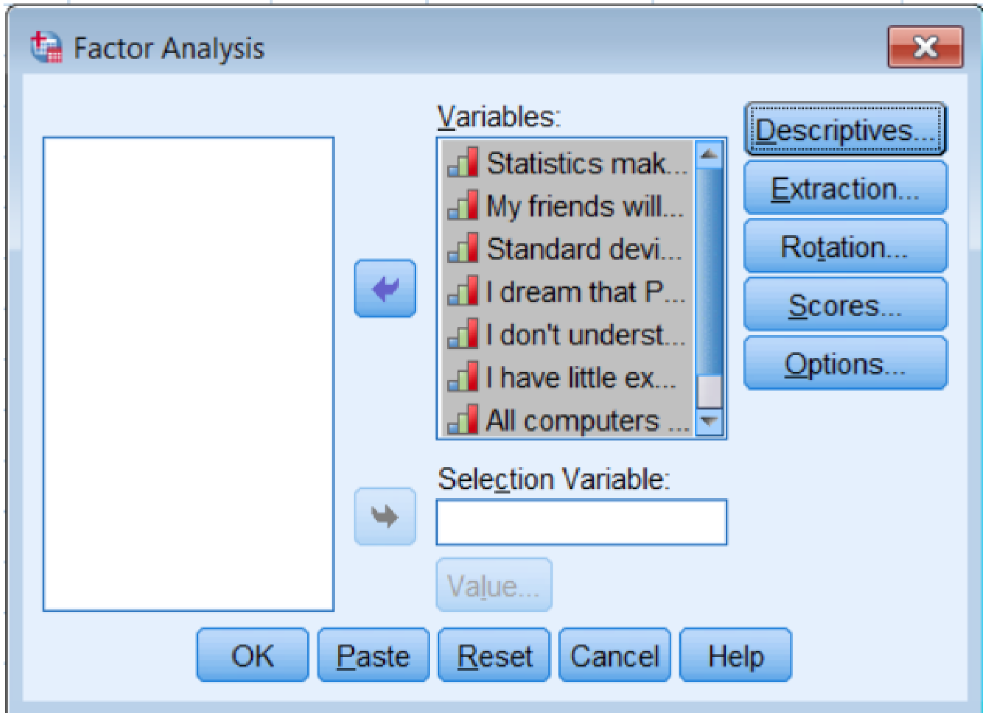


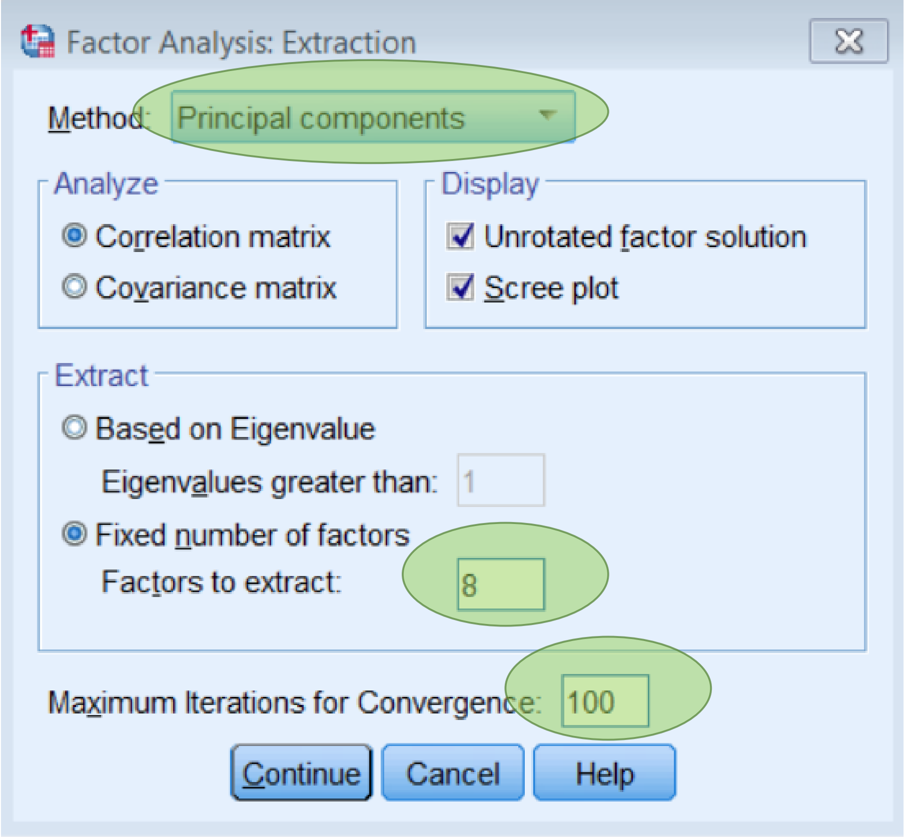
8 variables

2 components

Running a PCA with 8 components

Analyze – Dimension Reduction – Factor





Note: Factors = Components
8 components is NOT what you typically want to use

Component Matrix of the 8-component PCA

Component loadings

correlation of each item with the principal component

Component Matrix[a]

	Component							
	1	2	3	4	5	6	7	8
Statistics makes me cry	.659	.136	-.398	.160	-.064	.568	-.177	.068
My friends will think I'm stupid for not being able to cope with SPSS	-.300	.866	-.025	.092	-.290	-.170	-.193	-.001
Standard deviations excite me	-.653	.409	.081	.064	.410	.254	.378	.142
I dream that Pearson is attacking me with correlation coefficients	.720	.119	-.192	.064	-.288	-.089	.563	-.137
I don't understand statistics	.650	.096	-.215	.460	.443	-.326	-.092	-.010
I have little experience of computers	.572	.185	.675	.031	.107	.176	-.058	-.369
All computers hate me	.718	.044	.453	-.006	-.090	-.051	.025	.516
I have never been good at mathematics	.568	.267	-.221	-.694	.258	-.084	-.043	-.012
	3.057	1.067	0.958	0.736	0.622	0.571	0.543	0.446

Extraction Method: Principal Component Analysis.

a. 8 components extracted.

Sum of squared loadings across components is the communality

Q: why is it 1?

$0.659^2 = 0.434$

43.4% of the variance explained by first component

$0.136^2 = 0.018$

1.8% of the variance explained by second component

Sum squared loadings down each column (component) = eigenvalues

Total Variance Explained in the 8-component PCA

3.057 1.067 0.958 0.736 0.622 0.571 0.543 0.446

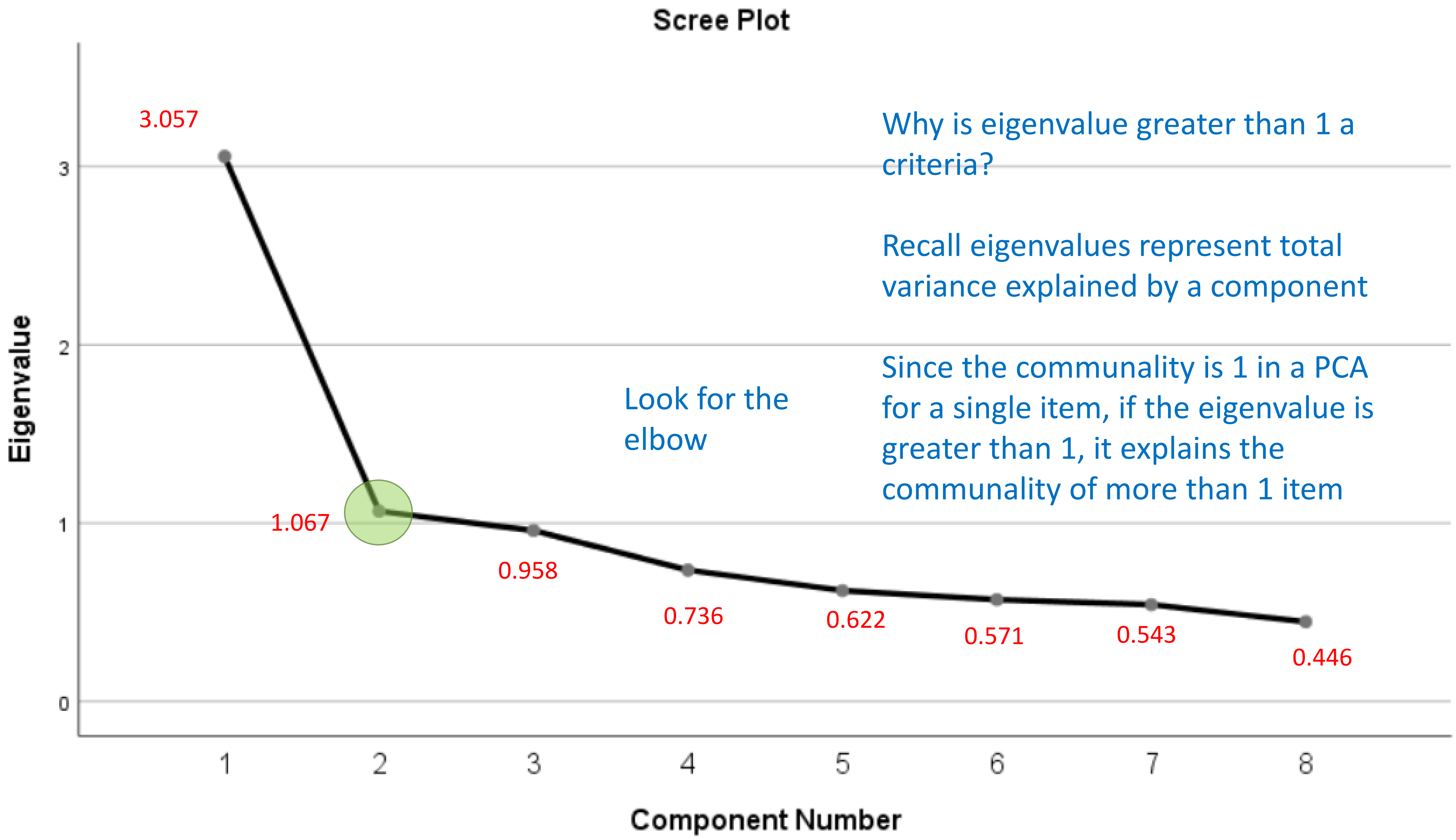
Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.057	38.206	38.206	3.057	38.206	38.206
2	1.067	13.336	51.543	1.067	13.336	51.543
3	.958	11.980	63.523	.958	11.980	63.523
4	.736	9.205	72.728	.736	9.205	72.728
5	.622	7.770	80.498	.622	7.770	80.498
6	.571	7.135	87.632	.571	7.135	87.632
7	.543	6.788	94.420	.543	6.788	94.420
8	.446	5.580	100.000	.446	5.580	100.000

Extraction Method: Principal Component Analysis.

Look familiar? Extraction Sums of Squared Loadings = Eigenvalues

Choosing the number of components to extract



Look for the elbow

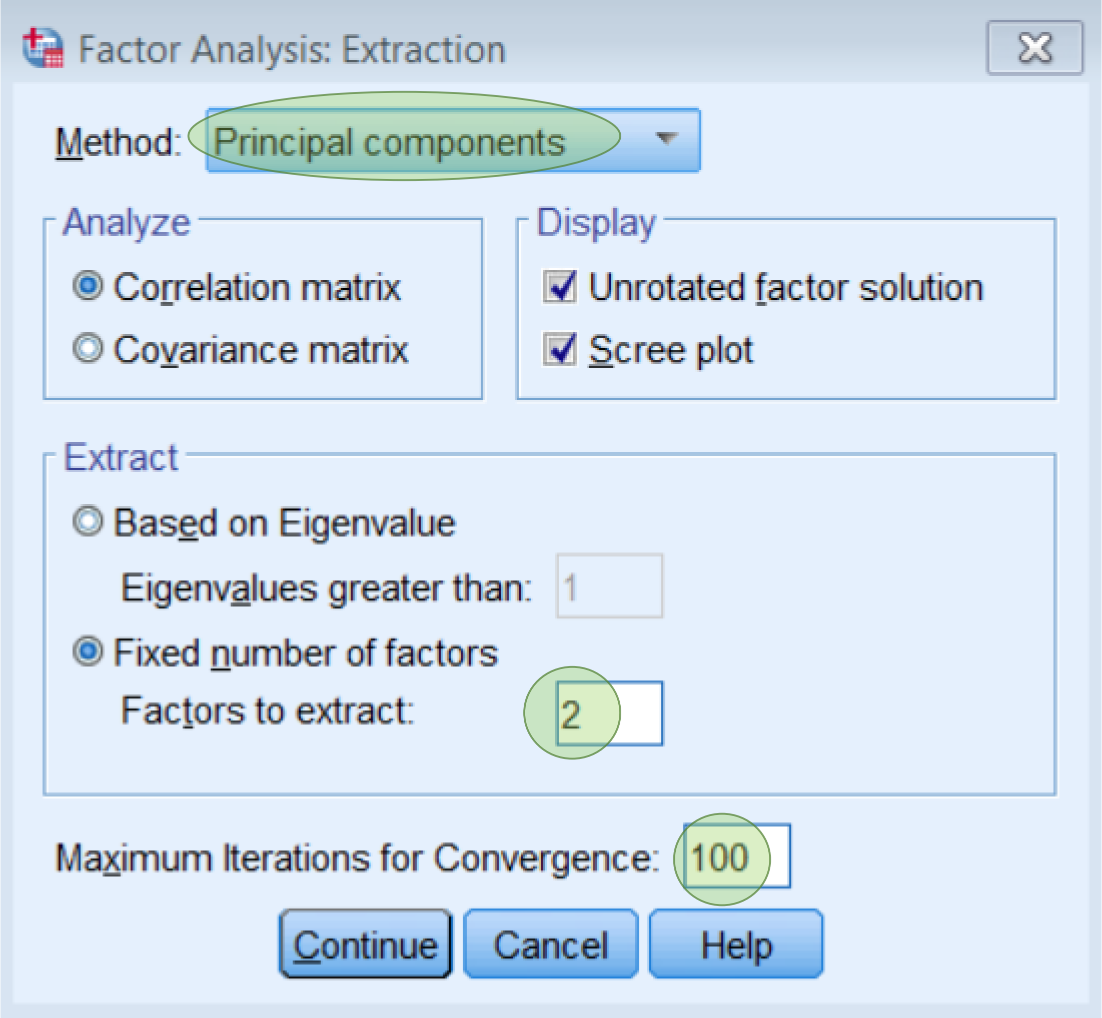
Why is eigenvalue greater than 1 a criteria?

Recall eigenvalues represent total variance explained by a component

Since the communality is 1 in a PCA for a single item, if the eigenvalue is greater than 1, it explains the communality of more than 1 item

Running a PCA with 2 components

Analyze – Dimension Reduction – Factor



Goal of PCA is dimension reduction

This is more realistic than an 8-component solution

Output from 2-Component PCA

Recall these numbers from the 8-component solution

3.057 1.067 0.958 0.736 0.622 0.571 0.543 0.446

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.057	38.206	38.206	3.057	38.206	38.206
2	1.067	13.336	51.543	1.067	13.336	51.543
3	.958	11.980	63.523			
4	.736	9.205	72.728			
5	.622	7.770	80.498			
6	.571	7.135	87.632			
7	.543	6.788	94.420			
8	.446	5.580	100.000			

Extraction Method: Principal Component Analysis.

Notice only two eigenvalues

Notice communalities not equal 1

Communalities

	Initial	Extraction
Statistics makes me cry	1.000	.453
My friends will think I'm stupid for not being able to cope with SPSS	1.000	.840
Standard deviations excite me	1.000	.594
I dream that Pearson is attacking me with correlation coefficients	1.000	.532
I don't understand statistics	1.000	.431
I have little experience of computers	1.000	.361
All computers hate me	1.000	.517
I have never been good at mathematics	1.000	.394

Extraction Method: Principal Component Analysis.

How would you derive these communalities?

Extracting Factors

- Principal components analysis
  - PCA with 8 / 2 components
- Common factor analysis
  - Principal axis factoring (2-factor PAF)
  - Maximum likelihood (2-factor ML)

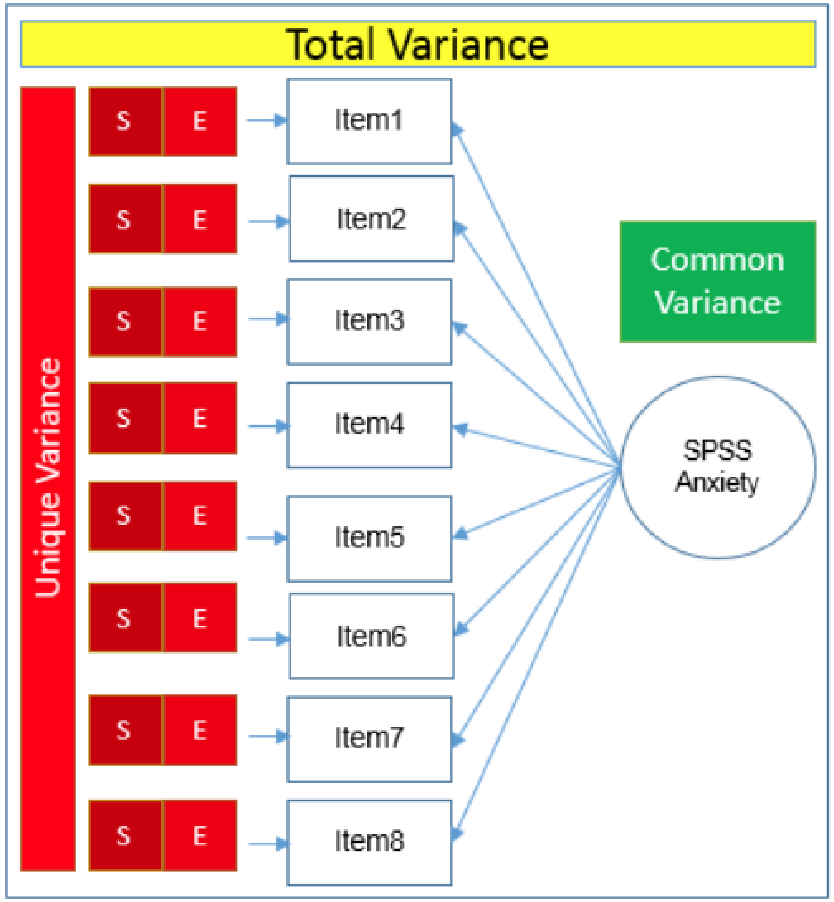
Factor Analysis

- **Factor Analysis (EFA)**
  - Goal: also to reduce dimensionality, BUT assume total variance can be divided into common and unique variance
    - Makes more sense to define a **construct** with measurement error

Correlations

	Statistics makes me cry	My friends will think I'm stupid for not being able to cope with SPSS	Standard deviations excite me	I dream that Pearson is attacking me with correlation coefficients	I don't understand statistics	I have little experience of computers	All computers hate me	I have never been good at mathematics
Statistics makes me cry	1							
My friends will think I'm stupid for not being able to cope with SPSS	-.099	1						
Standard deviations excite me	-.337	.318	1					
I dream that Pearson is attacking me with correlation coefficients	.436	-.112	-.380	1				
I don't understand statistics	.402	-.119	-.310	.401	1			
I have little experience of computers	.217	-.074	-.227	.278	.257	1		
All computers hate me	.305	-.159	**-.382**	.409	.339	**.514**	1	
I have never been good at mathematics	.331	-.050	-.259	.349	.269	.223	.297	1

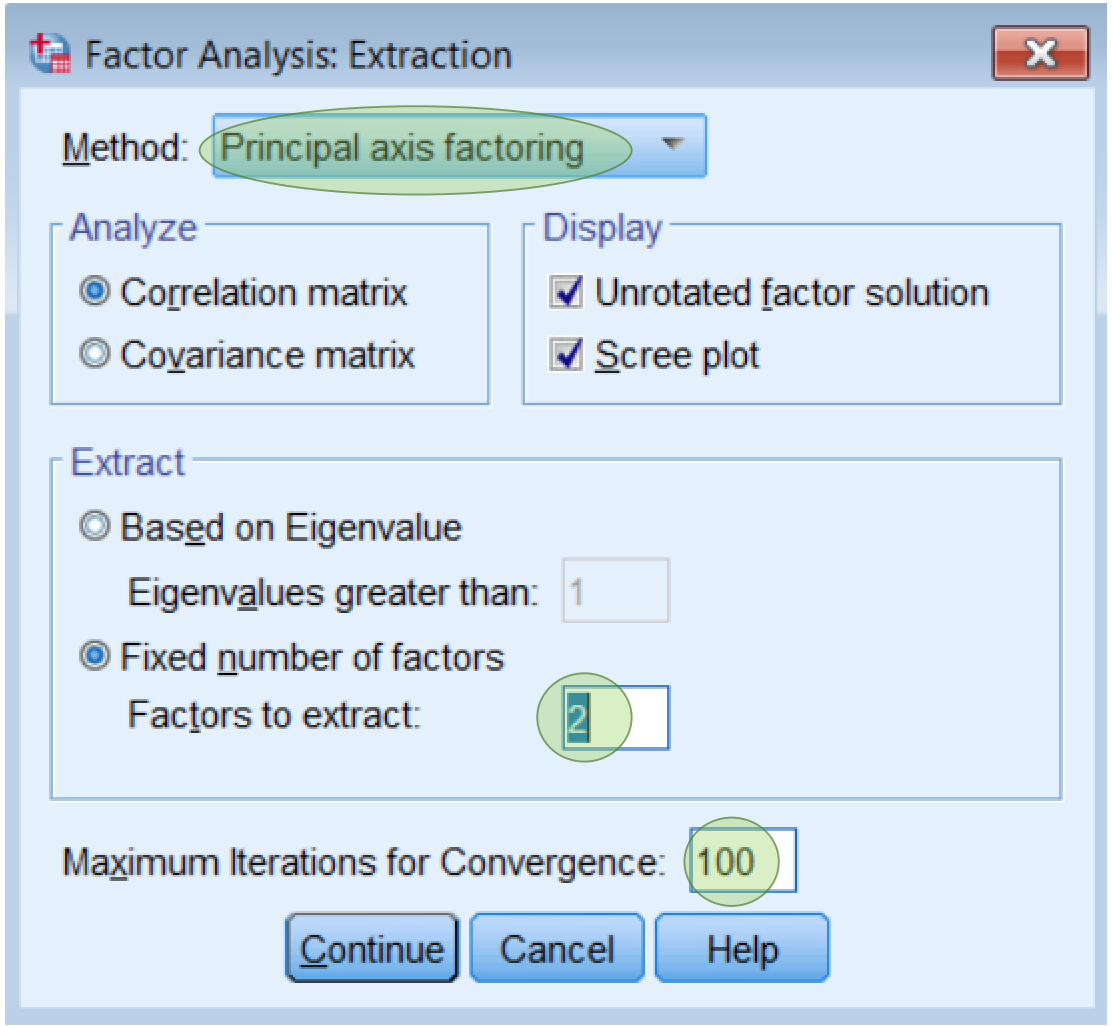
8 variables



1 variable = factor

Common Factor Analysis with 2 factors (PAF)

Analyze – Dimension Reduction – Factor



Make note of the word *eigenvalue* it will come back to haunt us later

SPSS does not change its menu to reflect changes in your analysis. You have to know the idiosyncrasies yourself.

Communalities of the 2-factor PAF

Initial communalities are the squared multiple correlation coefficients controlling for all other items in your model

Q: what was the initial communality for PCA?

Communalities

	Initial	Extraction
Statistics makes me cry	.293	.437
My friends will think I'm stupid for not being able to cope with SPSS	.106	.052
Standard deviations excite me	.298	.319
I dream that Pearson is attacking me with correlation coefficients	.344	.460
I don't understand statistics	.263	.344
I have little experience of computers	.277	.309
All computers hate me	.393	.851
I have never been good at mathematics	.192	.236

Extraction Method: Principal Axis Factoring.

Sum of squared loadings = 3.01

Total Variance Explained (2-factor PAF)

Unlike the PCA model, the sum of the initial eigenvalues do not equal the sums of squared loadings

Sum eigenvalues = 4.124

The reason is because Eigenvalues are for PCA not for factor analysis! (SPSS idiosyncrasies)

Total Variance Explained

Factor	Initial Eigenvalues Total	Initial Eigenvalues % of Variance	Initial Eigenvalues Cumulative %	Extraction Sums of Squared Loadings Total	Extraction Sums of Squared Loadings % of Variance	Extraction Sums of Squared Loadings Cumulative %
1	3.057	38.206	38.206	2.511	31.382	31.382
2	1.067	13.336	51.543	.499	6.238	37.621
3	.958	11.980	63.523			
4	.736	9.205	72.728			
5	.622	7.770	80.498			
6	.571	7.135	87.632			
7	.543	6.788	94.420			
8	.446	5.580	100.000			

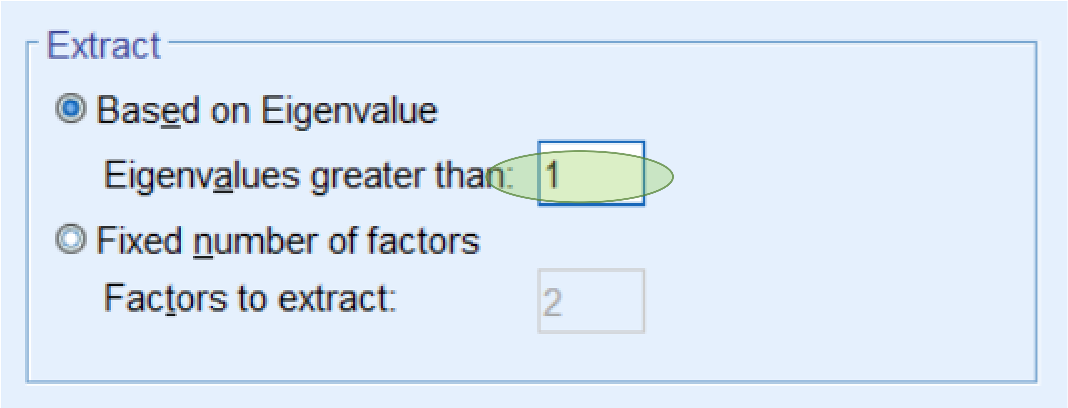
Extraction Method: Principal Axis Factoring.

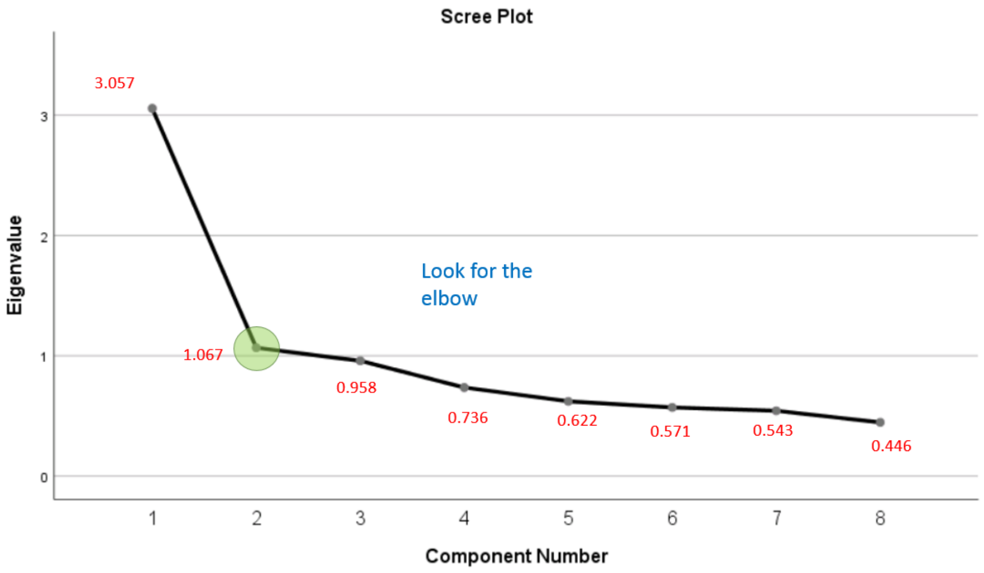
2.510 0.499

Sum of squared loadings = 3.01

Eigenvalues do not belong in EFA!

Analyze – Dimension Reduction – Factor





Caution!

Eigenvalues are only for PCA, yet SPSS uses the eigenvalue criteria for EFA

When you look at the scree plot in SPSS, you are making a conscious decision to use the PCA solution as a proxy for your EFA

Factor Matrix (2-factor PAF)

Squaring the loadings and summing up gives you either the *Communality* or the *Extraction Sums of Squared Loadings*

Summing down the communalities or across the eigenvalues gives you **total variance explained (3.01)**

These are analogous to component loadings in PCA

Factor Matrix[a]

	Factor 1	Factor 2		Communalities Extraction
Statistics makes me cry	.588	-.303	0.438	.437
My friends will think I'm stupid for not being able to cope with SPSS	-.227	.020	0.052	.052
Standard deviations excite me	-.557	.094	0.319	.319
I dream that Pearson is attacking me with correlation coefficients	.652	-.189	0.461	.460
I don't understand statistics	.560	-.174	0.344	.344
I have little experience of computers	.498	.247	0.309	.309
All computers hate me	.771	.506	0.850	.851
I have never been good at mathematics	.470	-.124	0.236	.236
	2.510	0.499	**3.01**	

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 79 iterations required.

Sum of squared loadings across factors is the communality

$0.588^2 = 0.346$

34.5% of the variance in Item 1 explained by first factor

$(-0.303)^2 = 0.091$

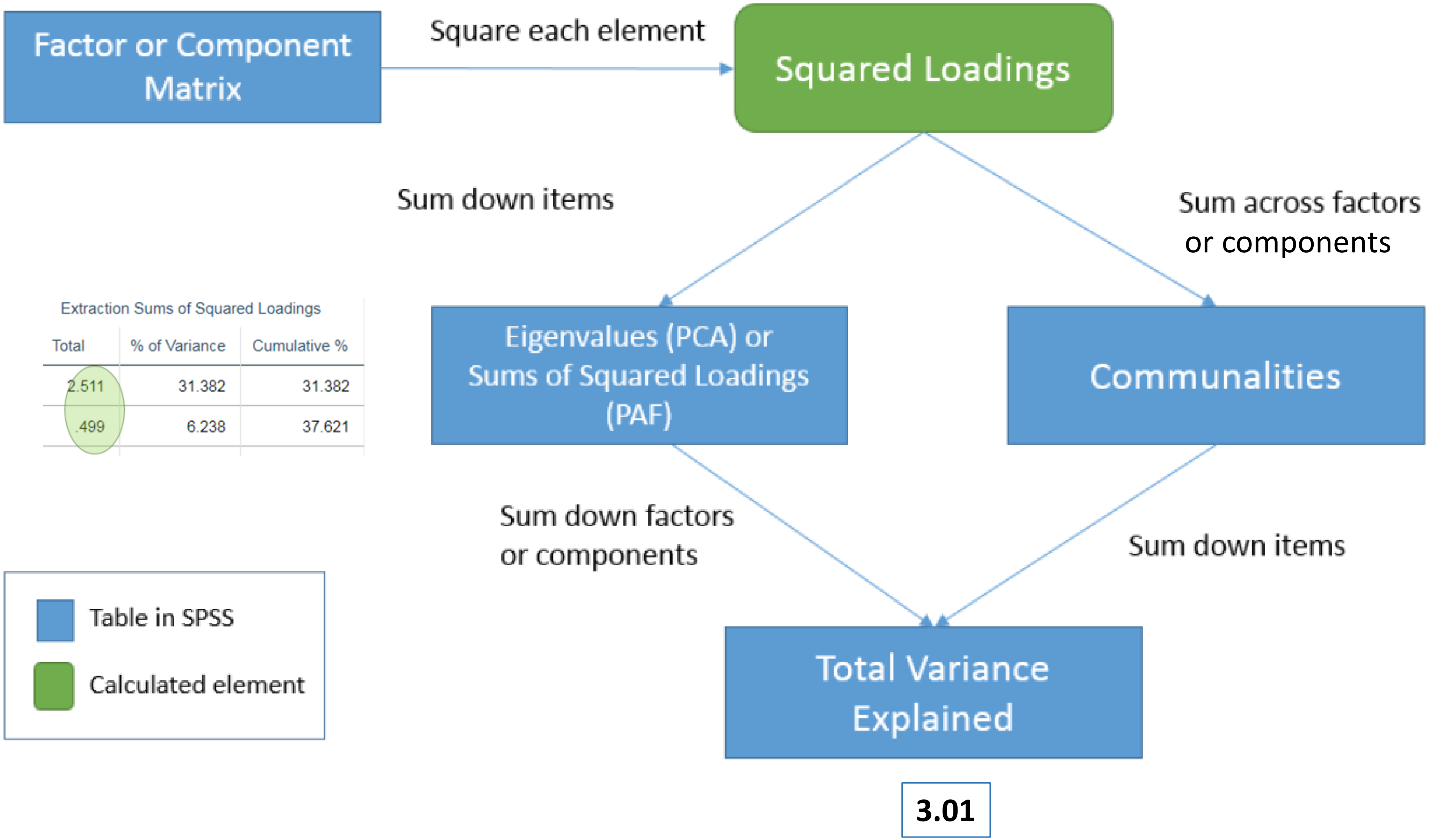
9.1% of the variance in Item 1 explained by second factor

$0.345 + 0.091 = 0.437$

43.7% of the variance in Item 1 explained by both factor = COMMUNALITY!

Sum squared loadings down each column = Extraction Sums of Square Loadings (not eigenvalues)

The relationship between the three tables



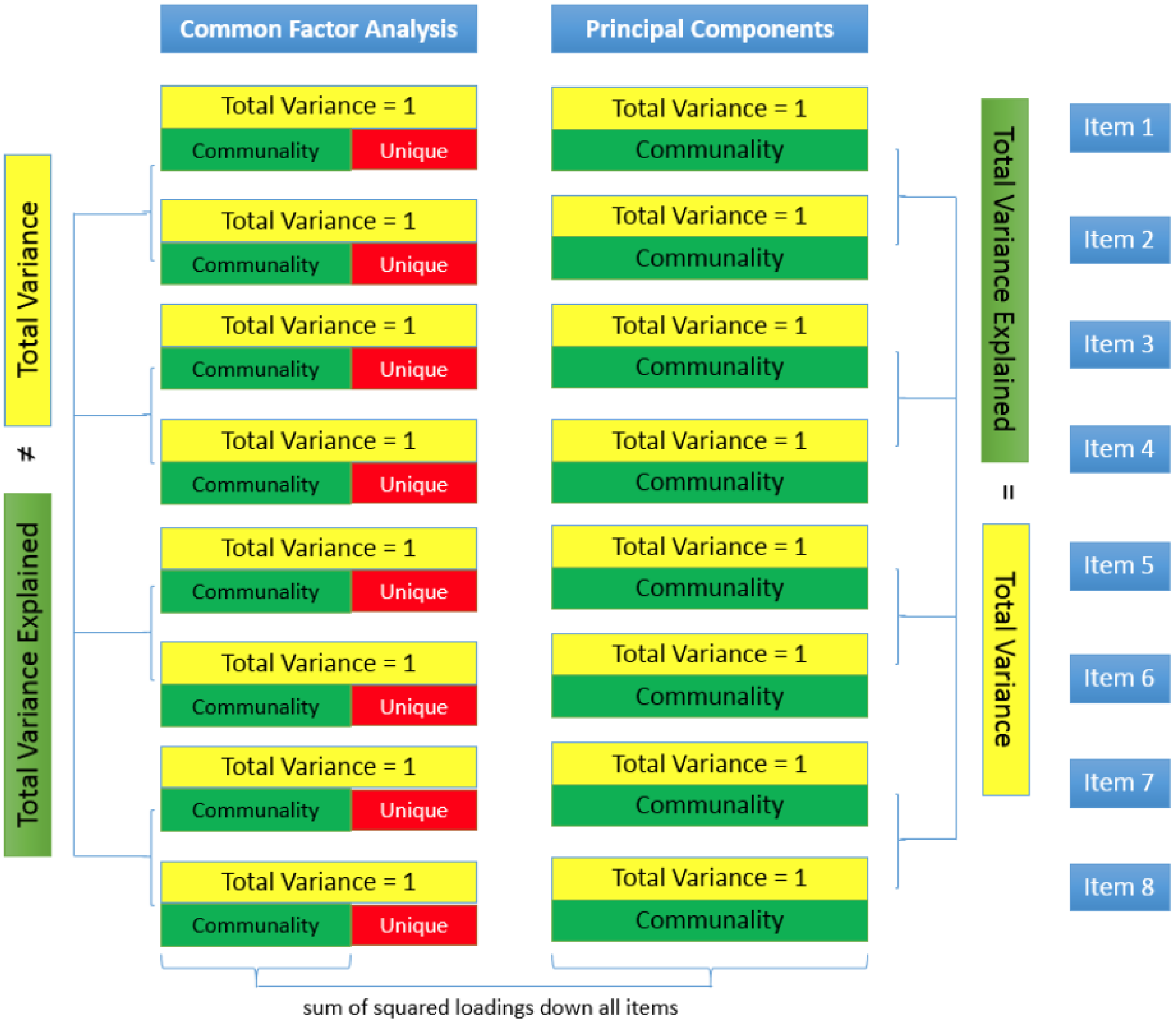
Extraction Sums of Squared Loadings		
Total	% of Variance	Cumulative %
2.511	31.382	31.382
.499	6.238	37.621

Extraction
.437
.052
.319
.460
.344
.309
.851
.236

Comparing EFA with PCA

EFA: Total Variance Explained = Total Communality Explained NOT Total Variance

For both models, communality is the total proportion of variance due to all factors or components in the model

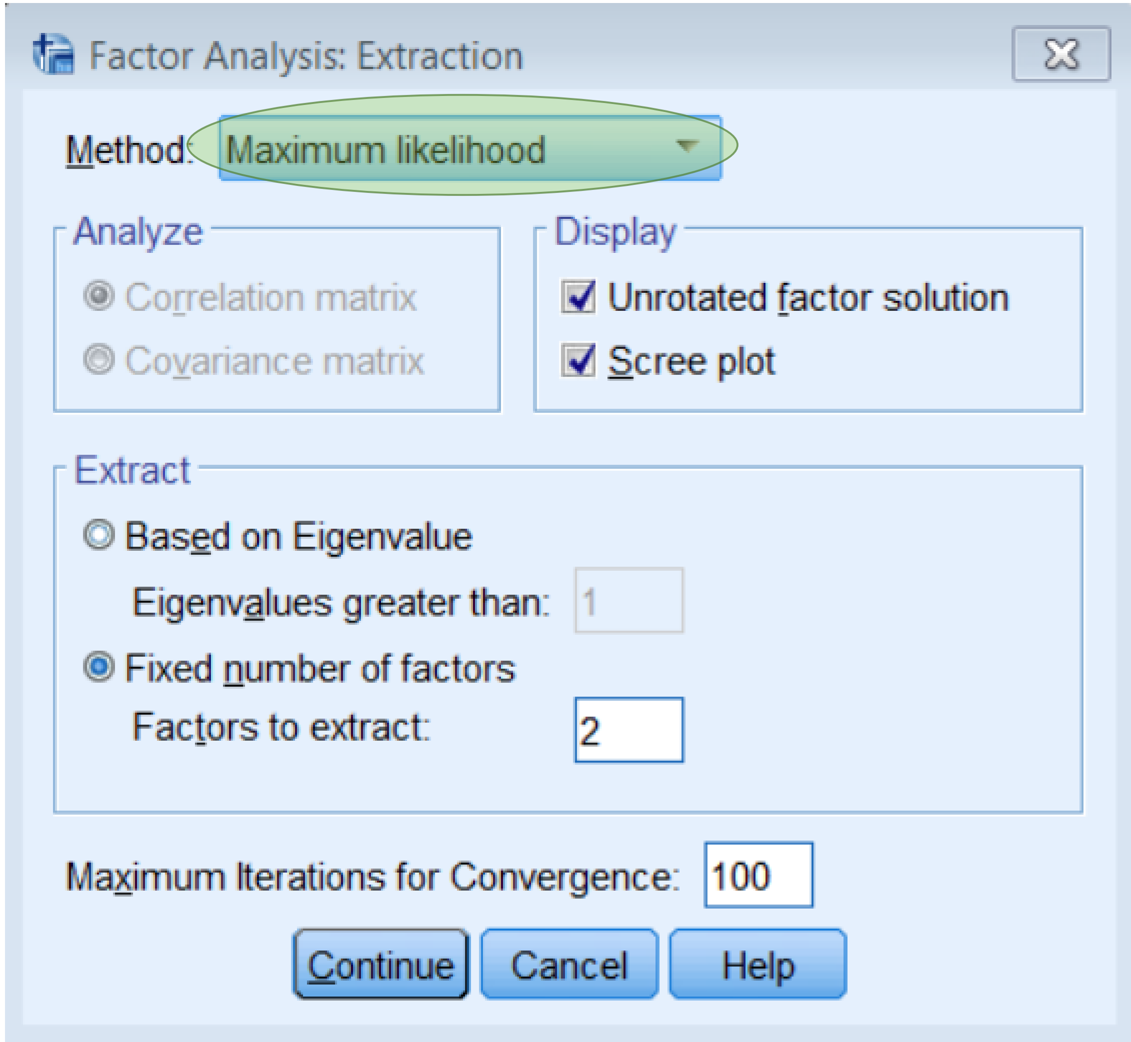


PCA: Total Variance Explained = Total Variance

Communalities are item specific

Maximum Likelihood Estimation (2-factor ML)

Analyze – Dimension Reduction – Factor



New output

A significant chi-square means you *reject* the current hypothesized model

Goodness-of-fit Test

Chi-Square	df	Sig.
198.617	13	.000

This is telling us we reject the two-factor model

Chi-square Comparison Table

Number of Factors	Chi-square	Df	p-value	Iterations needed
1	553.08	20	<0.05	4
2	198.62	13	< 0.05	39
3	13.81	7	0.055	57
4	1.386	2	0.5	168
5	NS	-2	NS	NS
6	NS	-5	NS	NS
7	NS	-7	NS	NS
8	N/A	N/A	N/A	N/A

Chi-square and degrees of freedom goes down

The three factor model is preferred from chi-square

Want NON-significant chi-square

Iterations needed goes up

An eight factor model is not possible in SPSS

Warnings

You cannot request as many factors as variables with any extraction method except PC. The number of factors will be reduced by one.
The number of degrees of freedom (-7) is not positive. Factor analysis may not be appropriate.

Rotation Methods

- Simple Structure
- Orthogonal rotation (Varimax)
- Oblique (Direct Oblimin)

Simple structure

1. Each item has high loadings on one factor only
2. Each factor has high loadings for only some of the items.

Pedhazur and Schemlkin (1991)

Item	Factor 1	Factor 2	Factor 3
1	0.8	0	0
2	0.8	0	0
3	0.8	0	0
4	0	0.8	0
5	0	0.8	0
6	0	0.8	0
7	0	0	0.8
8	0	0	0.8

The goal of rotation is to achieve simple structure

NOT simple structure

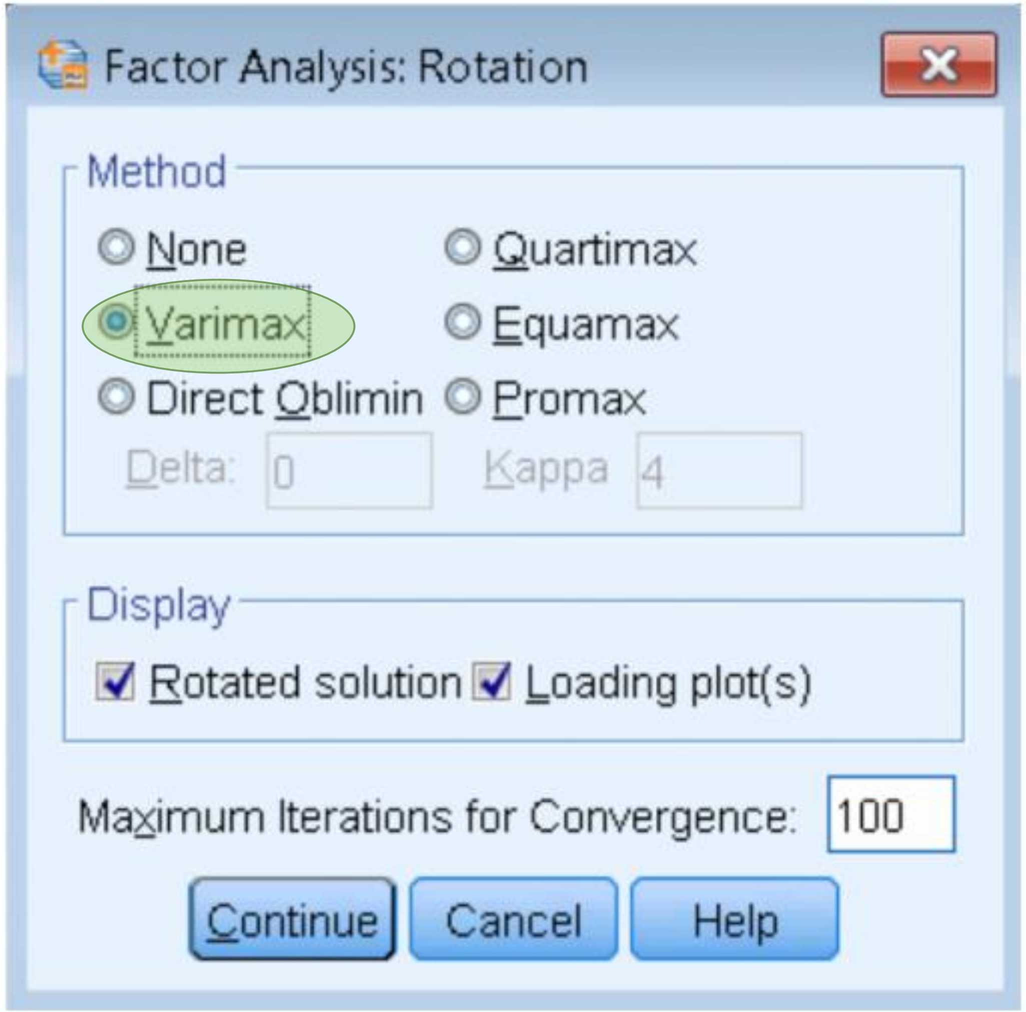
1. Most items have high loadings on *more* than one factor
2. Factor 3 has high loadings on 5/8 items

Item	Factor 1	Factor 2	Factor 3
1	0.8	0	0.8
2	0.8	0	0.8
3	0.8	0	0
4	0.8	0	0
5	0	0.8	0.8
6	0	0.8	0.8
7	0	0.8	0.8
8	0	0.8	0

Running a 2-factor solution (PAF Varimax rotation)

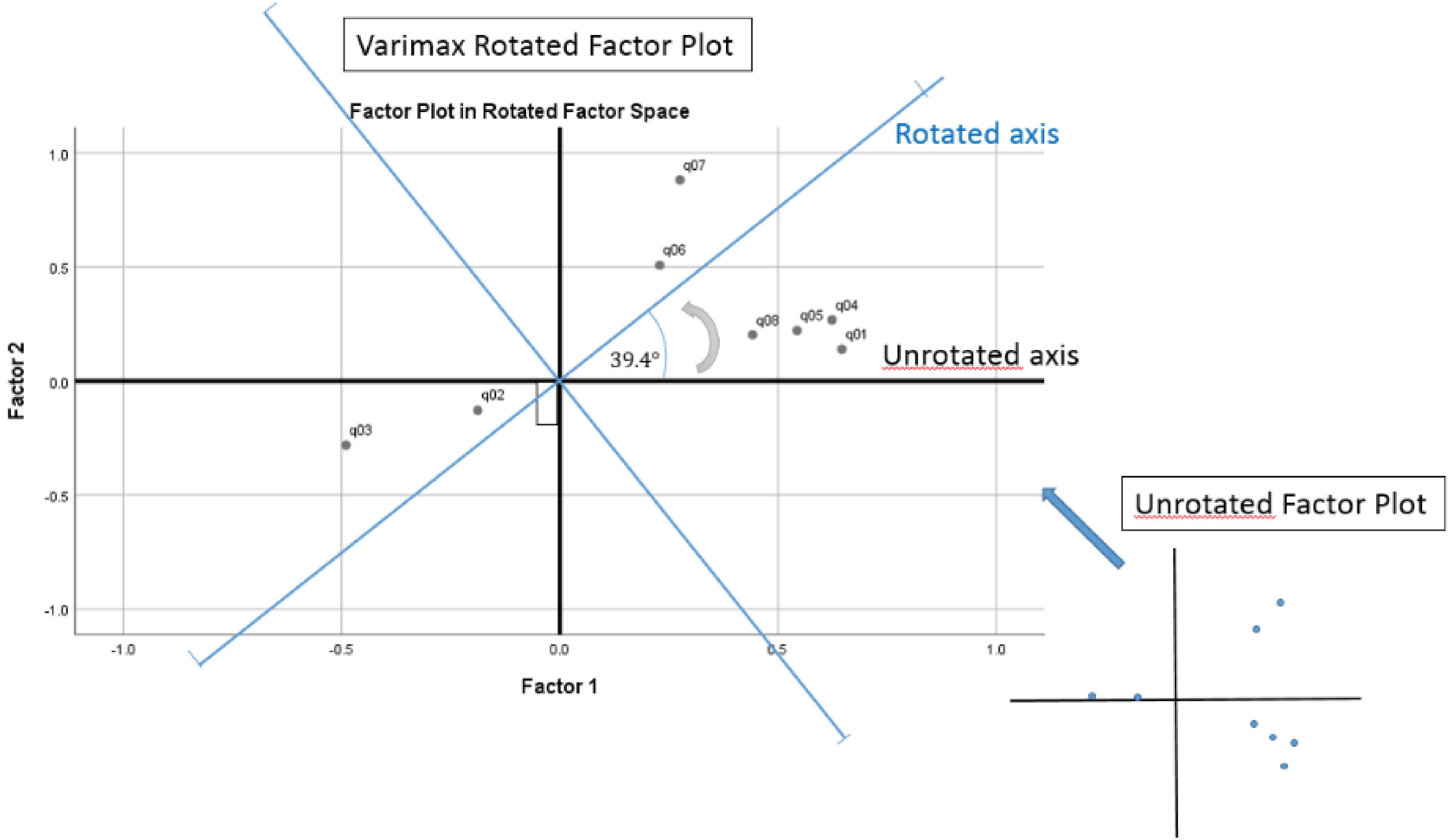
Varimax:
orthogonal rotation

maximizes variances of the loadings within the factors while maximizing differences between high and low loadings on a particular factor



Orthogonal means the factors are uncorrelated

Factor Loading Plot



Factor Transformation Matrix

The factor transformation matrix turns the regular factor matrix into the rotated factor matrix

Factor Transformation Matrix

Factor	1	2
1	.773	.635
2	-.635	.773

Extraction Method: Principal Axis Factoring. Rotation Method: Varimax with Kaiser Normalization.

The amount of rotation is the angle of rotation

Rotated Factor Matrix (2-factor PAF Varimax)

Factor Matrix[a]

Unrotated solution

	Factor 1	Factor 2	Communalities
Statistics makes me cry	.588	-.303	0.438
My friends will think I'm stupid for not being able to cope with SPSS	-.227	.020	0.052
Standard deviations excite me	-.557	.094	0.319
I dream that Pearson is attacking me with correlation coefficients	.652	-.189	0.461
I don't understand statistics	.560	-.174	0.344
I have little experience of computers	.498	.247	0.309
All computers hate me	.771	.506	0.850
I have never been good at mathematics	.470	-.124	0.236

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 79 iterations required.

Rotated Factor Matrix[a]

Varimax rotation

	Factor 1	Factor 2	Communalities
Statistics makes me cry	.646	.139	0.437
My friends will think I'm stupid for not being able to cope with SPSS	-.188	-.129	0.052
Standard deviations excite me	-.490	-.281	0.319
I dream that Pearson is attacking me with correlation coefficients	.624	.268	0.461
I don't understand statistics	.544	.221	0.344
I have little experience of computers	.229	.507	0.309
All computers hate me	.275	.881	0.850
I have never been good at mathematics	.442	.202	0.236

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

maximizes differences between high and low loadings on a particular factor

Notice that communalities are the same

Total Variance Explained (2-factor PAF Varimax)

True or False: Rotation changes how the variances are distributed but not the total communality

Total Variance Explained

Factor	Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.511	31.382	31.382	1.521	19.010	19.010
2	.499	6.238	37.621	1.489	18.610	37.621

Extraction Method: Principal Axis Factoring.

3.01

3.01

maximizes variances of the loadings

Even though the distribution of the variance is different the total sum of squared loadings is the same

Answer: T

Varimax vs. Quartimax

Quartimax: maximizes the squared loadings so that each item loads most strongly onto a single factor.
Good for generating a single factor.

Total Variance Explained

	Rotation Sums of Squared Loadings (Varimax)			Rotation Sums of Squared Loadings (Quartimax)		
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.521	19.010	19.010	2.381	29.760	29.760
2	1.489	18.610	37.621	.629	7.861	37.621

Extraction Method: Principal Axis Factoring.

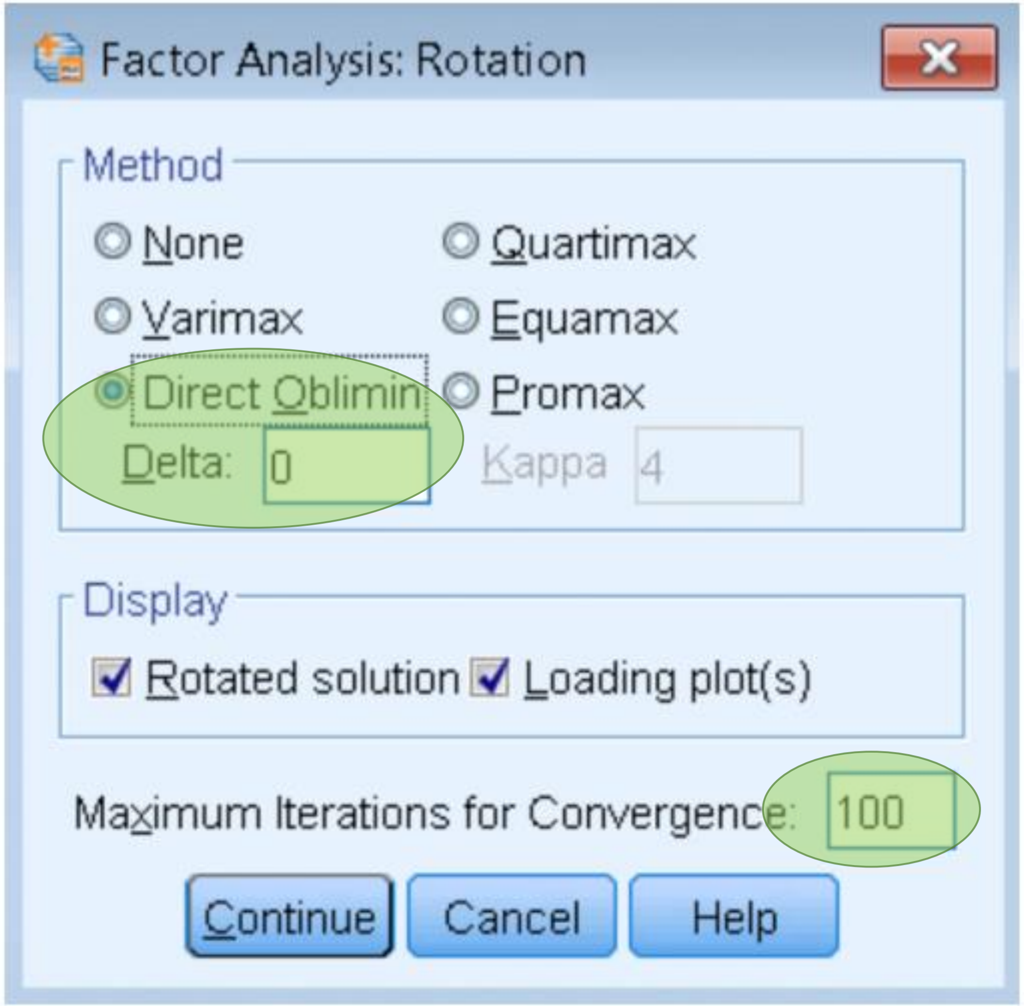
Varimax: good for distributing among more than one factor

Oblique Rotation

- **factor pattern matrix**
  - partial standardized regression coefficients of each item with a particular factor
- **factor structure matrix**
  - simple zero order correlations of each item with a particular factor
- **factor correlation matrix**
  - matrix of intercorrelations among factors

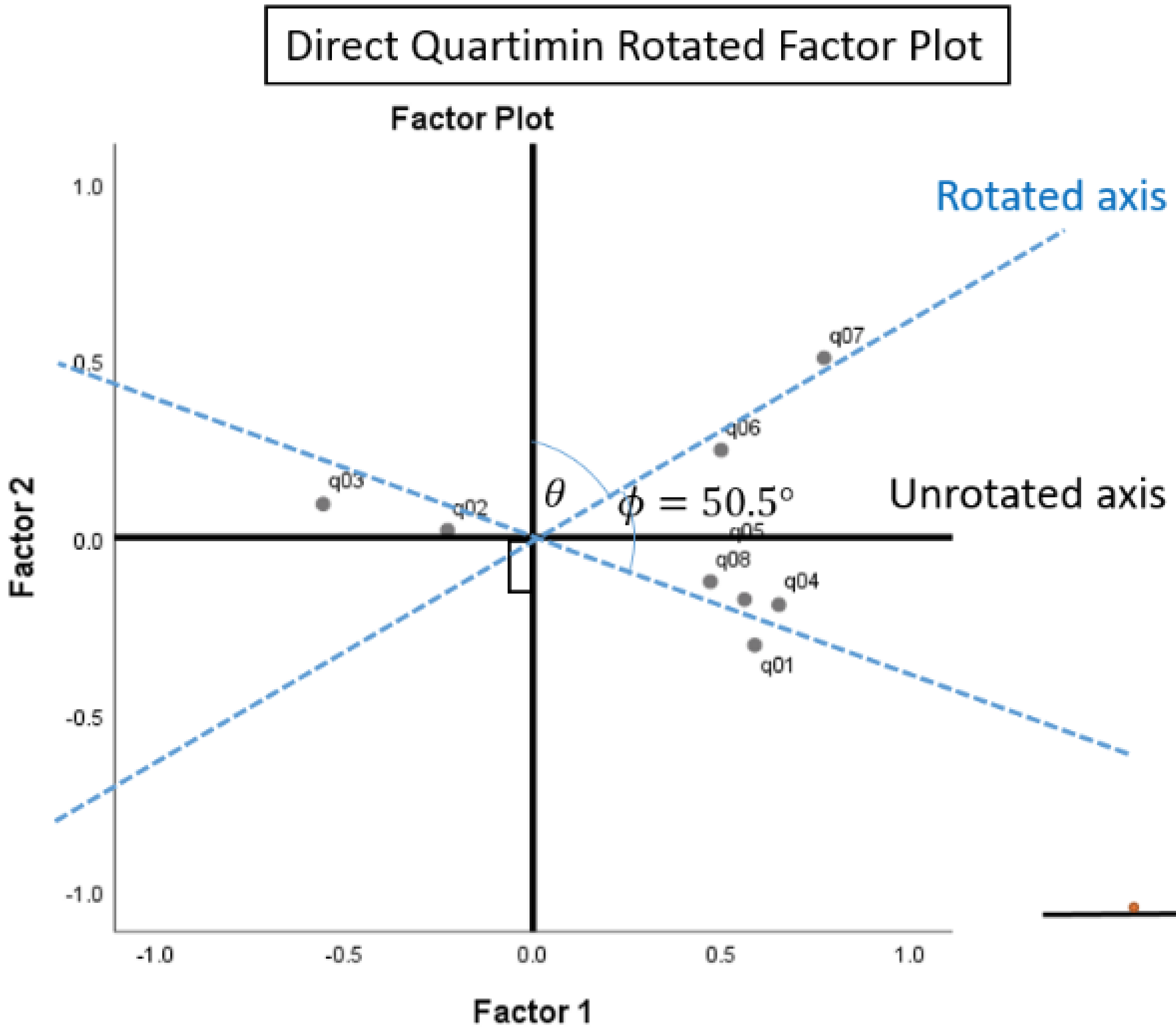
Running a two-factor solution (PAF) with Quartimin

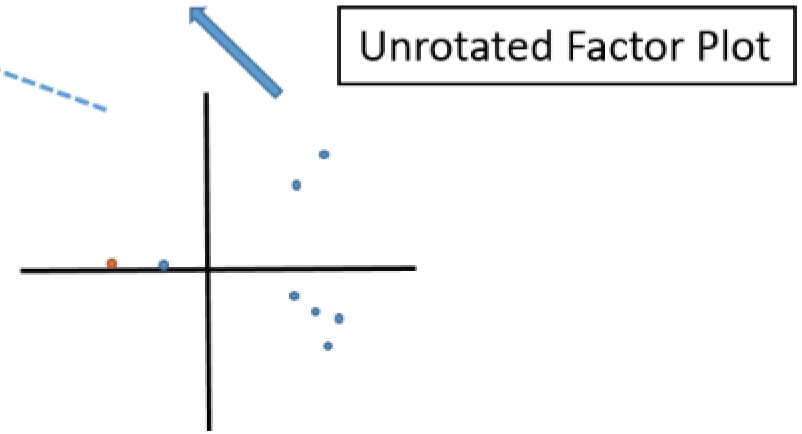
When Delta =0 →
Direct Quartimin



Oblique rotation means the factors are correlated

Factor plot of Direct Quartimin Rotation





angle of correlation φ
determines whether the factors are orthogonal or oblique

angle of axis rotation θ
how the axis rotates in relation to the data points (analogous to rotation in orthogonal rotation)

Factor Correlation Matrix (2-factor PAF Quartimin)

The more correlated the factors, the greater the difference between pattern and structure matrix

Factor Correlation Matrix

Factor	1	2
1	1.000	.636
2	.636	1.000

Extraction Method: Principal Axis Factoring. Rotation Method: Oblimin with Kaiser Normalization.

If the factors are orthogonal, the correlations between them would be zero, then the factor pattern matrix would EQUAL the factor structure matrix.

Structure & Pattern Matrix (2-factor PAF Direct Quartimin)

Partial standardized regression coefficients (can exceed one)

Pattern Matrix[a]

	Factor 1	Factor 2
Statistics makes me cry	.740	-.137
My friends will think I'm stupid for not being able to cope with SPSS	-.180	-.067
Standard deviations excite me	-.490	-.108
I dream that Pearson is attacking me with correlation coefficients	.660	.029
I don't understand statistics	.580	.011
I have little experience of computers	.077	.504
All computers hate me	-.017	.933
I have never been good at mathematics	.462	.036

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

0.566
0.037
0.252
0.436
0.337
0.260
0.871
0.215

0.740 is the effect of Factor 1 on Item 1 *controlling* for Factor 2

There IS a way to make the sum of squared loadings equal to the communality. Think back to Orthogonal Rotation.

Structure Matrix

	Factor 1	Factor 2
Statistics makes me cry	.653	.333
My friends will think I'm stupid for not being able to cope with SPSS	-.222	-.181
Standard deviations excite me	-.559	-.420
I dream that Pearson is attacking me with correlation coefficients	.678	.449
I don't understand statistics	.587	.380
I have little experience of computers	.398	.553
All computers hate me	.577	.923
I have never been good at mathematics	.485	.330

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

0.537
0.082
0.489
0.661
0.489
0.464
1.185
0.344

Simple zero order correlations (can't exceed one)

0.653 is the simple correlation of Factor 1 on Item 1

Note that the sum of squared loadings do NOT match communalities

Communalities

	Initial	Extraction
Statistics makes me cry	.293	.437
My friends will think I'm stupid for not being able to cope with SPSS	.106	.052
Standard deviations excite me	.298	.319
I dream that Pearson is attacking me with correlation coefficients	.344	.460
I don't understand statistics	.263	.344
I have little experience of computers	.277	.309
All computers hate me	.393	.851
I have never been good at mathematics	.192	.236

Extraction Method: Principal Axis Factoring.

Total Variance Explained (2-factor PAF Quartimin)

This is exactly the same as the unrotated 2-factor PAF solution

SPSS uses the structure matrix to calculate this
-factor contributions will overlap and become greater than the total variance

Total Variance Explained

Factor	Extraction Sums of Squared Loadings Total	% of Variance	Cumulative %	Rotation Sums of Squared Loadings[a] Total
1	2.511	31.382	31.382	2.318
2	.499	6.238	37.621	1.931

Extraction Method: Principal Axis Factoring.

a. When factors are correlated, sums of squared loadings cannot be added to obtain a total variance.

3.01

4.25

Note: now the sum of the squared loadings is HIGHER than the unrotated solution

SPSS uses the structure matrix to calculate this
-factor contributions will overlap and become greater than the total variance

Factor or Pattern Matrix?

- There is no consensus about which one to use in the literature
- Hair et al. (1995)
  - Better to interpret the factor pattern matrix because it gives the unique contribution of the factor on a particular item
- Pett et al. (2003)
  - Structure matrix should be used for interpretation
  - Pattern matrix for obtaining factor scores
- My belief: I agree with Hair

Hair, J. F. J., Anderson, R. E., Tatham, R. L., & Black, W. C. (1995). Multivariate data analysis . Saddle River.

Pett, M. A., Lackey, N. R., & Sullivan, J. J. (2003). *Making sense of factor analysis: The use of factor analysis for instrument development in health care research*. Sage.

Interpreting loadings (2-factor PAF Quartimin)

Factor

Structure Matrix

Pattern Matrix

	Structure Matrix		Pattern Matrix	
	1	2	1	2
Statistics makes me cry	.653	.333	.740	-.137
My friends will think I'm stupid for not being able to cope with SPSS	-.222	-.181	-.180	-.067
Standard deviations excite me	-.559	-.420	-.490	-.108
I dream that Pearson is attacking me with correlation coefficients	.678	.449	.660	.029
I don't understand statistics	.587	.380	.580	.011
I have little experience of computers	.398	.553	.077	.504
All computers hate me	.577	.923	-.017	.933
I have never been good at mathematics	.485	.330	.462	.036

Extraction Method: Principal Axis Factoring. Rotation Method: Oblimin with Kaiser Normalization.

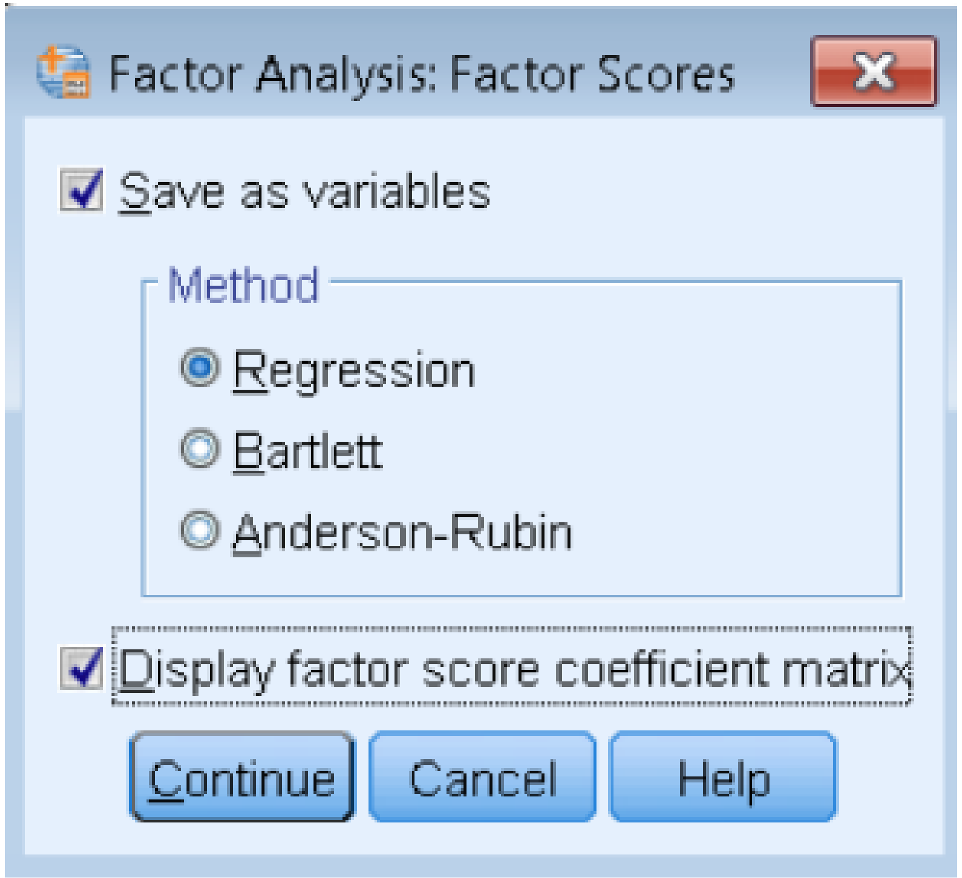
Why do you think the second loading is lower in the Pattern Matrix compared to the Structure Matrix?

Generating Factor Scores

- Regression
- Bartlett
- Anderson-Rubin

Generating factor scores (Quartimin, Reg Method)

Analyze – Dimension Reduction – Factor – Factor Scores



What it looks like in SPSS Data View

q08	FAC1_1	FAC2_1
1	-.87974	-.11277
2	-.93859	-.77805
2	-.31326	-.79102
2	1.07582	1.00312
2	-.52530	.04489

Factor Score Coefficient Matrix (Direct Quartimin)

This is how the factor scores are generated

SPSS takes the standardized scores for each item
Then multiply each score

Factor Score Coefficient Matrix

		Factor		
		1	2	
-0.452	Statistics makes me cry	.284	.005	-0.452
-0.733	My friends will think I'm stupid for not being able to cope with SPSS	-.048	-.019	-0.733
1.32	Standard deviations excite me	-.171	-.045	1.32
-0.829	I dream that Pearson is attacking me with correlation coefficients	.274	.045	-0.829
-0.749	I don't understand statistics	.197	.036	-0.749
-0.203	I have little experience of computers	.048	.095	-0.203
0.0692	All computers hate me	.174	.814	0.0692
-1.42	I have never been good at mathematics	.133	.028	-1.42
-0.880				**-0.113**

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization. Factor Scores Method: Regression.

Factor Score Covariance (Direct Quartimin)

Covariance matrix of the **true** factor scores

Factor Score

Covariance Matrix

Factor	1	2
1	1.897	1.895
2	1.895	1.990

Extraction Method: Principal Axis Factoring. Rotation Method: Oblimin with Kaiser Normalization. Factor Scores Method: Regression.

Covariance matrix of the **estimated** factor scores

Correlations

		REGR factor score 1 for analysis 1	REGR factor score 2 for analysis 1
REGR factor score 1 for analysis 1	Covariance	.777	.604
REGR factor score 2 for analysis 1	Covariance	.604	.870

Notice that for Direct Quartimin, the raw correlations do not match

Regression method has mean of zero, and variance equal to the squared multiple correlation of estimated and true factor scores

Factor Score Covariance (Varimax)

Factor Score Covariance Matrix

Factor	1	2
1	.831	.114
2	.114	.644

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax without Kaiser Normalization.
Factor Scores Method: Regression.

Correlations

		REGR factor score 1 for analysis 2	REGR factor score 2 for analysis 2
REGR factor score 1 for analysis 2	Covariance	.831	.114
REGR factor score 2 for analysis 2	Covariance	.114	.644

Notice that for Direct Quartimin, the raw correlations *do* match (property of Regression method)

However, note that the factor scores are still correlated even though we did Varimax

Choosing the right factor score

- Regression Method

Use if you want highest validity with your data and can sacrifice unbiasedness

  - mean of zero and variance = squared multiple correlation between **estimated** and **true** factor scores
  - can have factor correlation even when you choose Varimax rotation (biased)
  - highest validity with observed data

- Bartlett's Method

Use if you want unbiased scores

  - Unbiased estimate of true factor score
    - With repeated sampling, the average of the **estimated** scores equals the average of the **true** scores

- Anderson Rubin

Do not use for oblique rotations!

  - Imposes restriction that factor scores are uncorrelated with other *factors* as well with other estimated *factor scores*
    - will definitely get a correlation of zero

Correlations

		A-R factor score 1 for analysis 4	A-R factor score 2 for analysis 4
A-R factor score 1 for analysis 4	Pearson Correlation	1	.000
A-R factor score 2 for analysis 4	Pearson Correlation	.000	1