Multiple Populations

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Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- **Multiple populations**
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

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Multiple Population Growth Modeling

- Group as a dummy variable
- Multiple-group analysis
- Multiple-group analysis of randomized interventions

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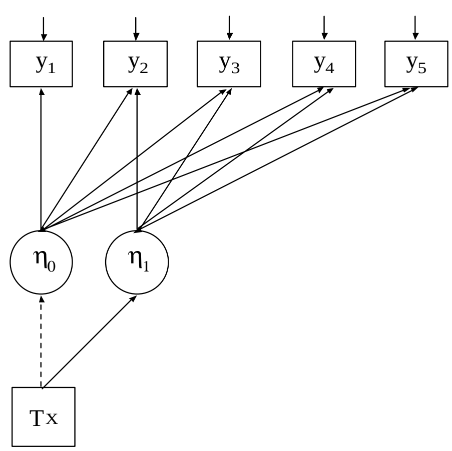
Group Dummy Variable As A Covariate



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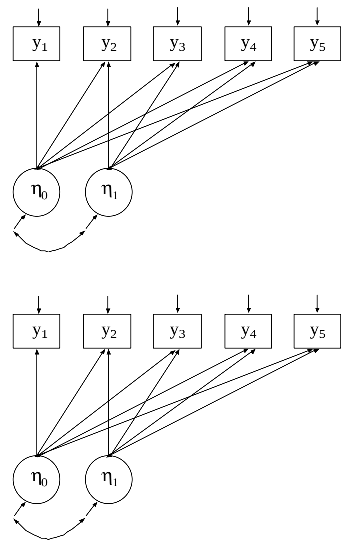
Two-Group Model



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Multiple Population Growth Modeling Specifications

Let y_{git} denote the outcome for population (group) g, individual i, and timepoint t,

$$\text{Level 1: } y_{gti} = \eta_{g0i} + \eta_{g1i}\, x_t + \varepsilon_{gti}\,, \qquad (65)$$
$$\text{Level 2a: } \eta_{g0i} = \alpha_{g0} + \gamma_{g0}\, w_{gi} + \zeta_{g0i}\,, \qquad (66)$$
$$\text{Level 2b: } \eta_{g1i} = \alpha_{g1} + \gamma_{g1}\, w_{gi} + \zeta_{g1i}\,, \qquad (67)$$

Measurement invariance (level-1 equation): time-invariant intercept 0 and slopes 1, x_t

Structural differences (level-2): α_g , γ_g , $V(\zeta_g)$

Alternative parameterization:

$$\text{Level 1: } y_{gti} = \nu + \eta_{g0i} + \eta_{g1i}\, x_t + \varepsilon_{gti}\,, \qquad (68)$$

with α_{10} fixed at zero in level 2a.

Analysis steps:

1. Separate growth analysis for each group
2. Joint analysis of all groups, free structural parameters
3. Join analysis of all groups, tests of structural parameter invariance

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NLSY: Multiple Cohort Structure

Birth Year Cohort	Age[a]																			
	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
57								82	83	84	85	86	87	88	89	90	91	92	93	94
58							82	83	84	85	86	87	88	89	90	91	92	93	94	
59						82	83	84	85	86	87	88	89	90	91	92	93	94		
60					82	83	84	85	86	87	88	89	90	91	92	93	94			
61				82	83	84	85	86	87	88	89	90	91	92	93	94				
62			82	83	84	85	86	87	88	89	90	91	92	93	94					
63		82	83	84	85	86	87	88	89	90	91	92	93	94						
64	82	83	84	85	86	87	88	89	90	91	92	93	94							

[a] Non-shaded areas represent years in which alcohol measures were obtained

Multiple Group Modeling Of Multiple Cohorts

- Data – two cohorts born in 1961 and 1962 measured on the frequency of heavy drinking in the years 1983, 1984, 1988, and 1989
- Development of heavy drinking across chronological age, not year of measurement, is of interest

Cohort/Year	1983	1984	1988	1989
1961 (older)	**22**	23	**2**7	28
1962 (younger)	21	**22**	26	**27**

Cohort/Age	21	22	23	24	25	26	27	28
1961 (older)		**83**	**84**				**88**	**89**
1962 (younger)	**83**	**84**				**88**	**89**	

Multiple Group Modeling Of Multiple Cohorts (Continued)

- Time scores calculated for age, not year of measurement

Age	**21**	**22**	**23**	24	25	**26**	**27**	**28**
Time score	**0**	**1**	**2**	3	4	**5**	**6**	**7**

Cohort 1961 time scores 1 2 6 7
Cohort 1962 time scores 0 1 5 6

- Can test the degree of measurement and structural invariance
  - Test of full invariance
    - Growth factor means, variances, and covariances held equal across cohorts
    - Residual variances of shared ages held equal across cohorts

Input For Multiple Group Modeling Of Multiple Cohorts

```
TITLE:       Multiple Group Modeling Of Multiple Cohorts
DATA:        FILE IS cohort.dat;
VARIABLE:    NAMES ARE cohort hd83 hd84 hd88 hd89;
             MISSING ARE *;
             USEV = hd83 hd84 hd88 hd89;
             GROUPING IS cohort (61 = older 62 = younger);
MODEL:       i s | hd83@0 hd84@1 hd88@5 hd89@6;
             [i] (1);
             [s] (2);
             i (3);
             s (4);
             i WITH s (5);
```

Input For Multiple Group Modeling Of Multiple Cohorts (Continued)

```
MODEL older:
              i s | hd83@1 hd84@2 hd88@6 hd89@7;
              hd83 (6);
              hd88 (7);
MODEL younger:
              hd84 (6);
              hd89 (7);

OUTPUT:       STANDARDIZED;
```

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Output Excerpts Multiple Group Modeling Of Multiple Cohorts

Tests Of Model Fit

```
Chi-Square Test of Model Fit
          Value                        68.096
          Degrees of Freedom               17
          P-Value                       .0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                        .047
          90 Percent C.I.                 .036   .059
```

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Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group OLDER					
I WITH					
S	-.111	.010	-11.390	-.537	-.537
Residual Variances					
HD83	1.141	.046	24.996	1.141	.445
HD84	1.062	.057	18.489	1.062	.453
HD88	1.028	.041	25.326	1.028	.455
HD89	.753	.053	14.107	.753	.358
Variances					
I	1.618	.068	23.651	1.000	1.000
S	.026	.002	13.372	1.000	1.000
Means					
I	1.054	.030	35.393	.828	.828
S	-.032	.005	-6.611	-.200	-.200

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Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

GROUP YOUNGER					
Residual Variances					
HD83	1.049	.066	15.916	1.049	.393
HD84	1.141	.046	24.996	1.141	.445
HD88	1.126	.056	19.924	1.126	.491
HD89	1.028	.041	25.326	1.028	.455

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Preventive Interventions Randomized Trials

Prevention Science Methodology Group (PSMG)

Developmental Epidemiological Framework:

- Determining the levels and variation in risk and protective factors as well as developmental paths within a defined population in the absence of intervention
- Directing interventions at these risk and protective factors in an effort to change the developmental trajectories in a defined population
- Evaluating variation in intervention impact across risk levels and contexts on proximal and distal outcomes, thereby empirically testing the developmental model

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Aggressive Classroom Behavior: The GBG Intervention

Muthén & Curran (1997, Psychological Methods)

The Johns Hopkins Prevention Center carried out a school-based preventive intervention randomized trial in Baltimore public schools starting in grade 1. One of the interventions tested was the Good Behavior Game intervention, a classroom based behavior management strategy promoting good behavior. It was designed specifically to reduce aggressive behavior of first graders and was aimed at longer term impact on aggression through middle school.

One first grade classroom in a school was randomly assigned to receive the Good Behavior Game intervention and another matched classroom in the school was treated as control. After an initial assessment in fall of first grade, the intervention was administered during the first two grades.

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Aggressive Classroom Behavior: The GBG Intervention (Continued)

The outcome variable of interest was teacher ratings (TOCA-R) of each child's aggressive behavior (breaks rules, harms property, fights, etc.) in the classroom through grades 1 – 6. Eight teacher ratings were made from fall and spring for the first two grades and every spring in grades 3 – 6.

The most important scientific question was whether the Good Behavior Game reduces the slope of the aggression trajectory across time. It was also of interest to know whether the intervention varies in impact for children who started out as high aggressive versus low aggressive.

Analyses in Muthén-Curran (1997) were based on data for 75 boys in the GBG group who stayed in the intervention condition for two years and 111 boys in the control group.

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The GBG Aggression Example: Analysis Results

Muthén & Curran (1997):

- Step 1: Control group analysis
- Step 2: Treatment group analysis
- Step 3: Two-group analysis w/out interactions
- Step 4: Two-group analysis with interactions
- Step 5: Sensitivity analysis of final model
- Step 6: Power analysis

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MUTHÉN AND CURRAN

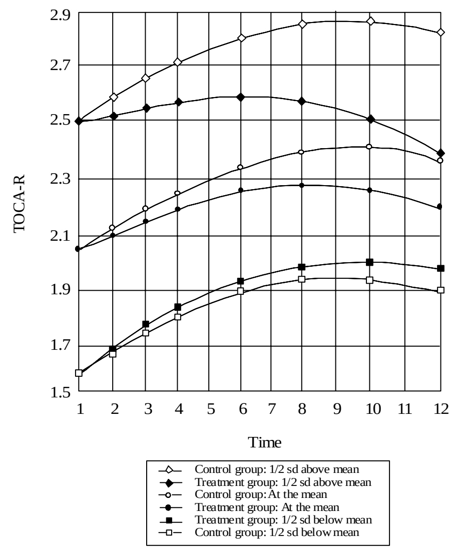

Figure 15. Model implied growth trajectories of Teacher Observation of Classroom Behavior—Revised (TOCA-R) scores as a function of initial status. Each timepoint represents one 6-month interval.

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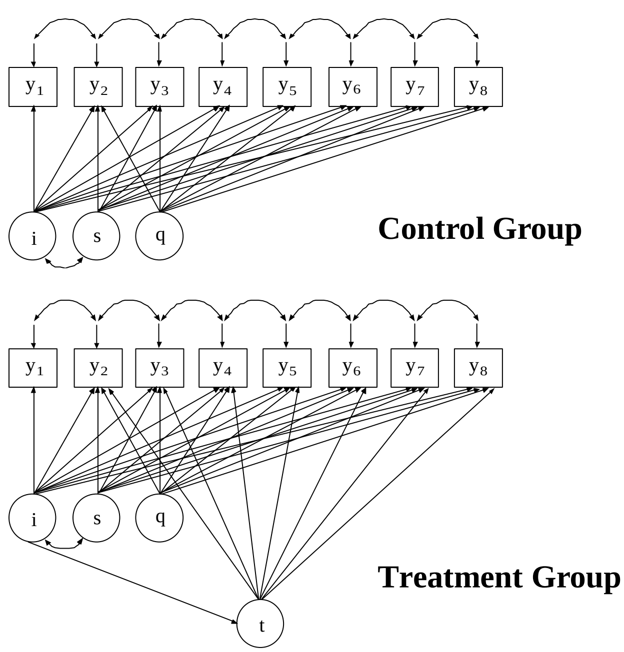

184

Input Excerpts For Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects

```
TITLE:          Aggressive behavior intervention growth model
                n = 111 for control group
                n = 75 for tx group
MODEL:          i s q | y1@0 y2@1 y3@2 y4@3 y5@5 y6@7 y7@9 y8@11;
                i t | y1@0 y2@1 y3@2 y4@3 y5@5 y6@7 y7@9 y8@11;
                [y1-y8] (1);   !alternative growth model
                [i@0];         !parameterization
                i (2);
                s (3);
                i WITH s (4);
                [s] (5);
                [q] (6);
                t@0 q@0;
                q WITH i@0 s@0 t@0; y1-y7 PWITH y2-y8;
                t ON i;
```

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Input Excerpts For Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

```
MODEL control:
                [s] (5);
                [q] (6);
                t ON i@0;
                [t@0];
```

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects

Tests Of Model Fit

```
Chi-Square Test of Model Fit

         Value                           64.553
         Degrees of Freedom                  50
         P-Value                          .0809

RMSEA (Root Mean Square Error Of Approximation)

         Estimate                          .056
         90 Percent C.I.                   .000   .092
```

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Control		Group Tx	
Observed Variable	R-Square	Observed Variable	R-Square
Y1	.644	Y1	.600
Y2	.642	Y2	.623
Y3	.663	Y3	.568
Y4	.615	Y4	.464
Y5	.637	Y5	.425
Y6	.703	Y6	.399
Y7	.812	Y7	.703
Y8	.818	Y8	.527

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Control

	Estimates	S.E.	Est./S.E.	Std	StdYX
T ON					
I	**.000**	**.000**	**.000**	**999.000**	**999.000**
Residual Variances					
Y1	.444	.088	5.056	.444	.356
Y2	.449	.079	5.714	.449	.358
Y3	.414	.069	6.026	.414	.337
Y4	.522	.080	6.551	.522	.385
Y5	.512	.079	6.469	.512	.363
Y6	.422	.074	5.677	.422	.297
Y7	.264	.083	3.186	.264	.188
Y8	.291	.094	3.097	.291	.182
T	.000	.000	.000	999.000	999.000
Variances					
I	.803	.109	7.330	1.000	1.000
S	.004	.001	3.869	1.000	1.000
Q	.000	.000	.000	999.000	999.000

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	.000	.000	.000	.000	.000
S	.086	.021	4.035	1.285	1.285
Q	-.005	.002	-3.005	999.000	999.000
Intercepts					
Y1	2.041	.078	26.020	2.041	1.828
Y2	2.041	.078	26.020	2.041	1.823
Y3	2.041	.078	26.020	2.041	1.841
Y4	2.041	.078	26.020	2.041	1.753
Y5	2.041	.078	26.020	2.041	1.718
Y6	2.041	.078	26.020	2.041	1.711
Y7	2.041	.078	26.020	2.041	1.724
Y8	2.041	.078	26.020	2.041	1.612
T	.000	.000	.000	999.000	999.000

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Tx

	Estimates	S.E.	Est./S.E.	Std	StdYX
T ON					
I	**-.052**	**.015**	**-3.347**	**-1.000**	**-1.000**
Residual Variances					
Y1	.535	.141	3.801	.535	.400
Y2	.439	.122	3.595	.439	.377
Y3	.501	.108	4.653	.501	.432
Y4	.701	.132	5.332	.701	.536
Y5	.736	.133	5.545	.736	.575
Y6	.805	.152	5.288	.805	.601
Y7	.245	.104	2.364	.245	.297
Y8	.609	.182	3.351	.609	.473
T	.000	.000	.000	.000	.000
Variances					
I	.803	.109	7.330	1.000	1.000
S	.004	.001	3.869	1.000	1.000
Q	.000	.000	.000	999.000	999.000

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Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	.000	.000	.000	.000	.000
S	.086	.021	4.035	1.285	1.285
Q	-.005	.002	-3.005	999.000	999.000
Intercepts					
Y1	2.041	.078	26.020	2.041	1.764
Y2	2.041	.078	26.020	2.041	1.893
Y3	2.041	.078	26.020	2.041	1.895
Y4	2.041	.078	26.020	2.041	1.785
Y5	2.041	.078	26.020	2.041	1.805
Y6	2.041	.078	26.020	2.041	1.764
Y7	2.041	.078	26.020	2.041	2.248
Y8	2.041	.078	26.020	2.041	1.799
T	-.016	.013	-1.225	-.341	-.341

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Growth Modeling With Multiple Indicators

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Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- **Multiple indicators**
- Embedded growth models
- Categorical latent variables: growth mixtures

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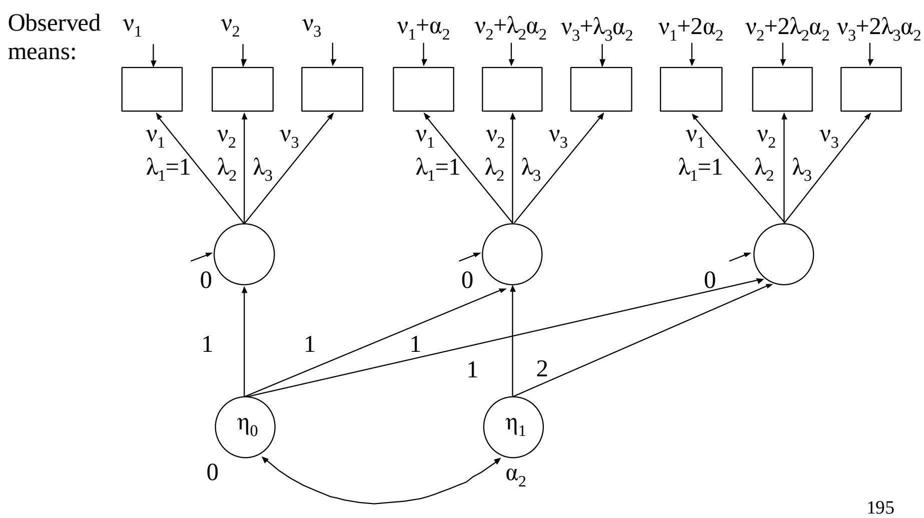
Growth Of Latent Variable Construct Measured By Multiple Indicators



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Multiple Indicator Growth Modeling Specifications

Let y_{jti} denote the outcome for individual i, indicator j, and timepoint t, and let η_{ti} denote a latent variable construct,

Level 1*a (measurement part):*

$$y_{jti} = \nu_{jt} + \lambda_{jt}\, \eta_{ti} + \varepsilon_{jti}\,, \qquad (44)$$

Level 1*b* : $\eta_{ti} = \eta_{0i} + \eta_{1i}\; x_t + \zeta_{ti}\,,$ (45)

Level 2*a* : $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}\,,$ (46)

Level 2*b* : $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}\,,$ (47)

Measurement invariance: time-invariant indicator intercepts and slopes:

$$\nu_{j1} = \nu_{j2} = \ldots \nu_{jT} = \nu_j\,, \qquad (48)$$

$$\lambda_{j1} = \lambda_{j2} = \ldots \lambda_{jT} = \lambda_j\,, \qquad (49)$$

where $\lambda_1 = 1$, $\alpha_0 = 0$. $V(\varepsilon_{jti})$ and $V(\zeta_{ti})$ may vary over time. Structural differences: $E(\eta_{ti})$ and $V(\eta_{ti})$ vary over time.

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Steps In Growth Modeling With Multiple Indicators

- Exploratory factor analysis of indicators for each timepoint
- Determine the shape of the growth curve for each indicator and the sum of the indicators
- Fit a growth model for each indicator—must be the same
- Confirmatory factor analysis of all timepoints together
  - Covariance structure analysis without measurement parameter invariance
  - Covariance structure analysis with invariant loadings
  - Mean and covariance structure analysis with invariant measurement intercepts and loadings
- Growth model with measurement invariance across timepoints

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Advantages Of Using Multiple Indicators Instead Of An Average

- Estimation of unequal weights
- Partial measurement invariance—changes across time in individual item functioning
- No confounding of time-specific variance and measurement error variance
- Smaller standard errors for growth factor parameters (more power)

198

Classroom Aggression Data (TOCA)

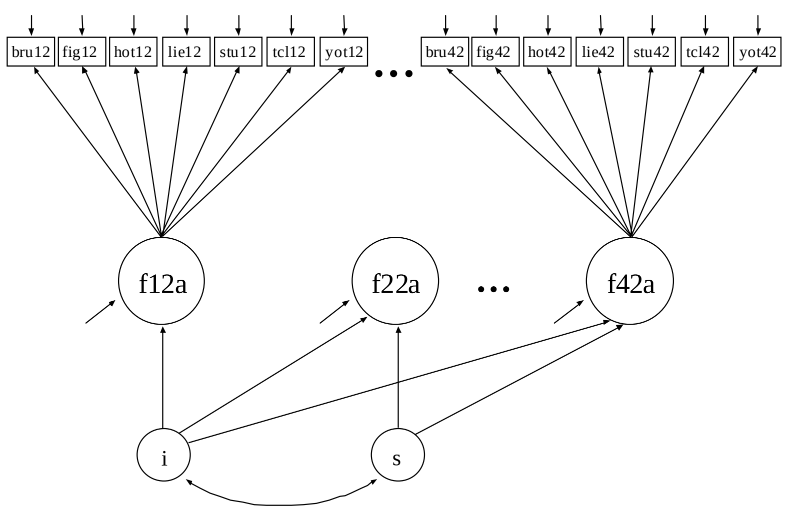
The classroom aggression data are from an intervention study in Baltimore public schools carried out by the Johns Hopkins Prevention Research Center. Subjects were randomized into treatment and control conditions. The TOCA-R instrument was used to measure 10 aggression items at multiple timpoints. The TOCA-R is a teacher rating of student behavior in the classroom. The items are rated on a six-point scale from almost never to almost always.

Data for this analysis include the 342 boys in the control group. Four time points are examined: Spring Grade 1, Spring Grade 2, Spring Grade 3, and Spring Grade 4.

Seven aggression items are used in the analysis:

- Break rules
- Fights
- Harms others
- Lies
- Stubborn
- Teasing classmates
- Yells at others

199



200

Input Excerpts For TOCA Data Multiple Indicator CFA With No Measurement Invariance

```
TITLE:       Multiple indicator CFA with no measurement invariance
.
.
.
MODEL:       f12a BY bru12
                     fig12
                     hot12
                     lie12
                     stu12
                     tcl12
                     yot12;

             f22a BY bru22
                     fig22
                     hot22
                     lie22
                     stu22
                     tcl22
                     yot22;
```

201

Input Excerpts For TOCA Data Multiple Indicator CFA With No Measurement Invariance (Continued)

```
MODEL:       f32a BY bru32
                     fig32
                     hot32
                     lie32
                     stu32
                     tcl32
                     yot32;

             f42a BY bru42
                     fig42
                     hot42
                     lie42
                     stu42
                     tcl42
                     yot42;
```

202

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance

```
TITLE:      Multiple indicator CFA with factor loading invariance
.
.
.
MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);

            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```

203

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance (Continued)

```
MODEL:      f32a BY bru32
                    fig32 (1)
                    hot32 (2)
                    lie32 (3)
                    stu32 (4)
                    tcl32 (5)
                    yot32 (6);

            f42a BY bru42
                    fig42 (1)
                    hot42 (2)
                    lie42 (3)
                    stu42 (4)
                    tcl42 (5)
                    yot42 (6);
```

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Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance

```
TITLE:      Multiple indicator CFA with factor loading and intercept
            incariance
.
.
.
MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);
            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```

205

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance (Continued)

```
MODEL:      f32a BY bru32
                    fig32 (1)
                    hot32 (2)
                    lie32 (3)
                    stu32 (4)
                    tcl32 (5)
                    yot32 (6);

            f42a BY bru42
                    fig42 (1)
                    hot42 (2)
                    lie42 (3)
                    stu42 (4)
                    tcl42 (5)
                    yot42 (6);
```

206

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32 hot42] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22 stu32 stu42] (11);
[tcl12 tcl22 tcl32 tcl42] (12);
[yot12 yot22 yot32 yot42] (13);

[f12a@0 f22a f32a f42a];
```

207

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance

```
TITLE:       Multiple indicator CFA with factor loading and partial
             intercept invariance

MODEL:       f12a BY bru12
                     fig12 (1)
                     hot12 (2)
                     lie12 (3)
                     stu12 (4)
                     tcl12 (5)
                     yot12 (6);
             f22a BY bru22
                     fig22 (1)
                     hot22 (2)
                     lie22 (3)
                     stu22 (4)
                     tcl22 (5)
                     yot22 (6);
```

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Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance (Continued)

```
f32a BY bru32
        fig32 (1)
        hot32 (2)
        lie32 (3)
        stu32 (4)
        tcl32 (5)
        yot32 (6);
f42a BY bru42
        fig42 (1)
        hot42 (2)
        lie42 (3)
        stu42 (4)
        tcl42 (5)
        yot42 (6);
```

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Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32      ] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22            ] (11);
[tcl12 tcl22 tcl32      ] (12);
[yot12 yot22 yot32 yot42] (13);

[f12a@0 f22a f32a f42a];
```

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Summary of Analysis Results For TOCA Measurement Invariance Models

Model	Chi-Square (d.f.)	Difference (d.f. diff.)
Measurement non-invariance	567.08 (344)	
Factor loading invariance	581.29 (362)	14.21 (18)
Factor loading and intercept invariance	654.59 (380)	73.30* (18)
Factor loading and partial intercept invariance	606.97 (376)	25.68* (14)
Factor loading and partial intercept invariance with a linear growth structure	614.74 (381)	7.77 (5)

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Summary of Analysis Results For TOCA Measurement Invariance Models (Continued)

Explanation of Chi-Square Differences

Factor loading invariance	(18)	6 factor loadings instead of 24
Factor loading and intercept invariance	(18)	7 intercepts plus 3 factor means instead of 28 intercepts
Factor loading and partial intercept invariance	(14)	4 additional intercepts
Factor loading and partial intercept invariance with a linear growth structure	(5)	1 growth factor mean instead of 3 factor means 2 growth factor variances, 1 growth factor covariance, 4 factor residual variances instead of 10 factor variances/covariances

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Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure

```
MODEL:      f12a BY bru12
                    fig12 (1)
                    hot12 (2)
                    lie12 (3)
                    stu12 (4)
                    tcl12 (5)
                    yot12 (6);
            f22a BY bru22
                    fig22 (1)
                    hot22 (2)
                    lie22 (3)
                    stu22 (4)
                    tcl22 (5)
                    yot22 (6);
```

213

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure (Continued)

```
MODEL:      f32a BY bru32
                    fig32 (1)
                    hot32 (2)
                    lie32 (3)
                    stu32 (4)
                    tcl32 (5)
                    yot32 (6);
            f42a BY bru42
                    fig42 (1)
                    hot42 (2)
                    lie42 (3)
                    stu42 (4)
                    tcl42 (5)
                    yot42 (6);
```

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Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure (Continued)

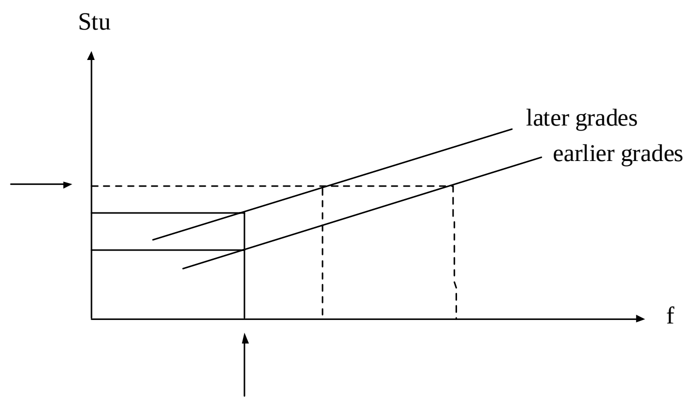
```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32      ] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22            ] (11);
[tcl12 tcl22 tcl32      ] (12);
[yot12 yot22 yot32 yot42] (13);

i s | f12a@0 f22a@1 f32a@2 f42a@3;
```

Alternative language:

```
i BY f12a-f42a@1;
s BY f12a@0 f22a@1 f32a@2 f42a@3;
[f12a-f42a@0 i@0 s];
```


Output Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure

	Estimates	S.E.	Est./S.E.	Std	StdYX
F12A |					
BRU12	1.000	.000	.000	.190	.786
FIG12	1.097	.039	28.425	.208	.868
HOT12	.986	.037	26.586	.187	.811
LIE12	.967	.041	23.769	.184	.742
STU12	.880	.041	21.393	.167	.667
TCL12	1.034	.039	26.206	.196	.786
YOT12	.932	.039	23.647	.177	.709
Intercepts					
STU12	.331	.013	25.408	.331	1.324
STU22	.331	.013	25.408	.331	1.231
STU32	.417	.017	24.345	.417	1.592
STU42	.390	.017	23.265	.390	1.496



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Degrees Of Invariance Across Time

- Case 1
  - Same items
  - All items invariant
  - Same construct
- Case 2
  - Same items
  - Some items non-invariant
  - Same construct
- Case 3
  - Different items
  - Some items invariant
  - Same construct
- Case 4
  - Different items
  - Some items invariant
  - Different construct

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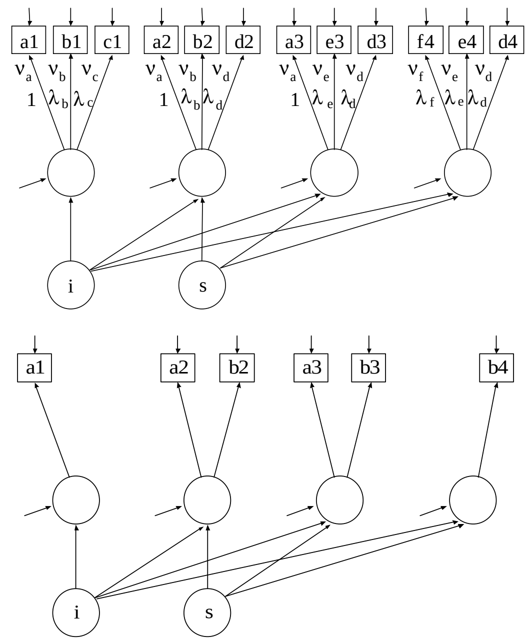

219

Embedded Growth Models

220

Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- Multiple indicators
- **Embedded growth models**
- Categorical latent variables: growth mixtures

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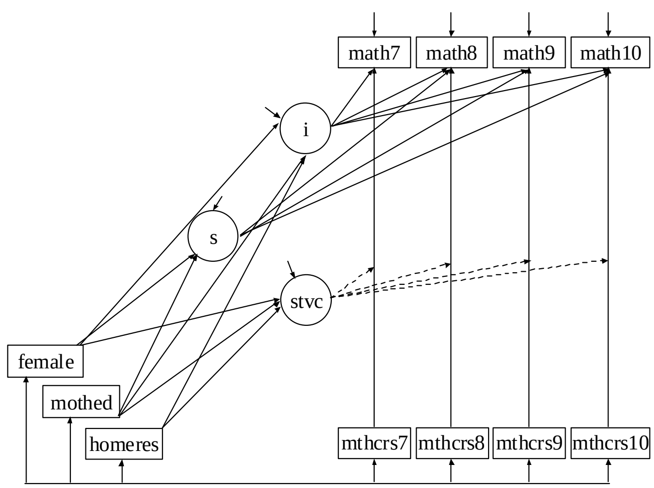
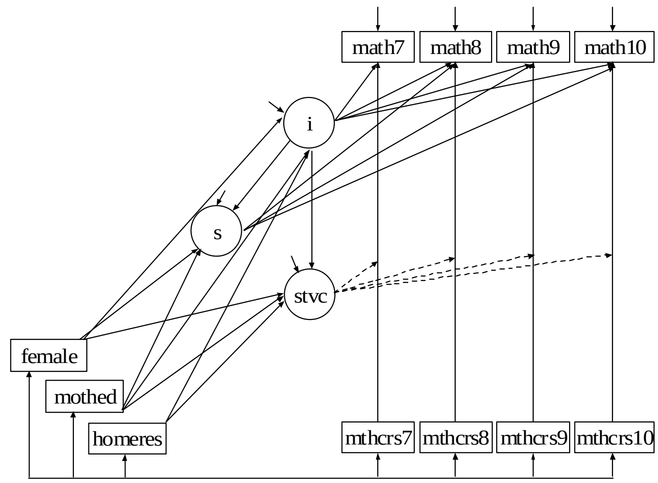
Growth Modeling With Time-Varying Covariates



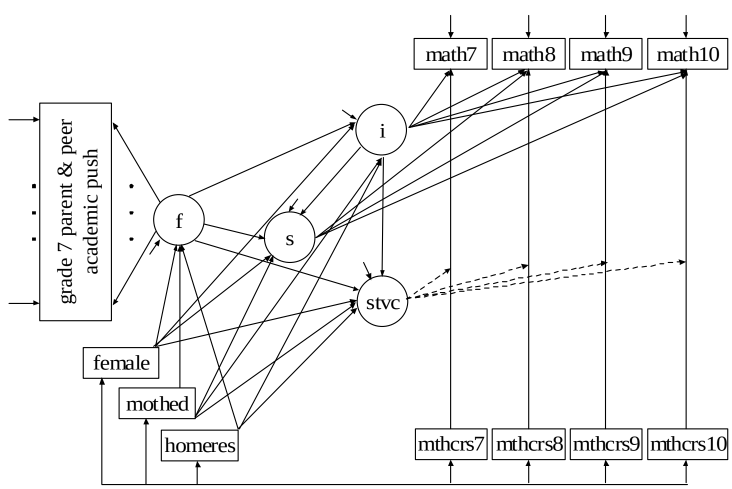
222

A Generalized Growth Model



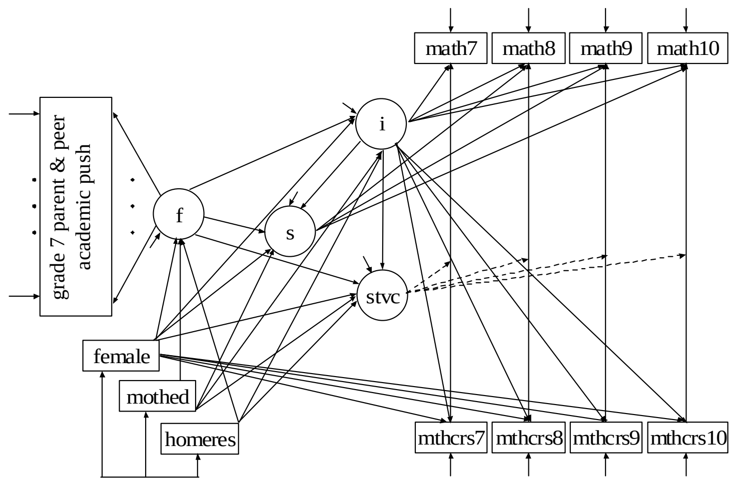
223

A Generalized Growth Model



224

A Generalized Growth Model



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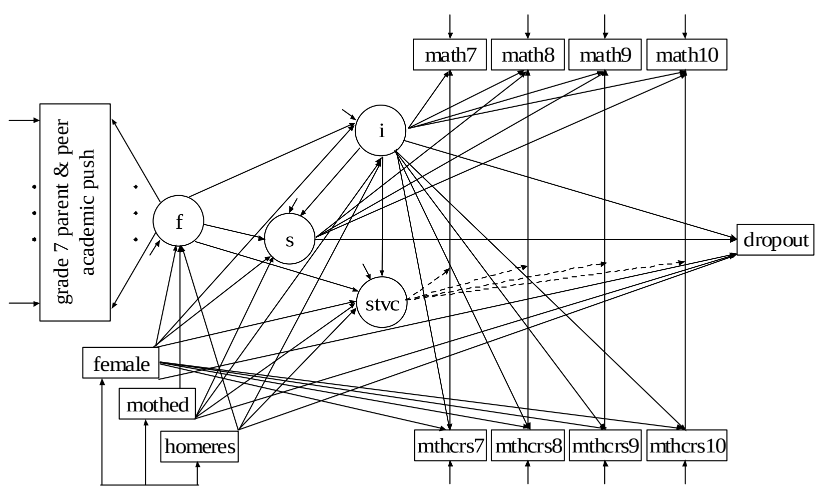
A Generalized Growth Model



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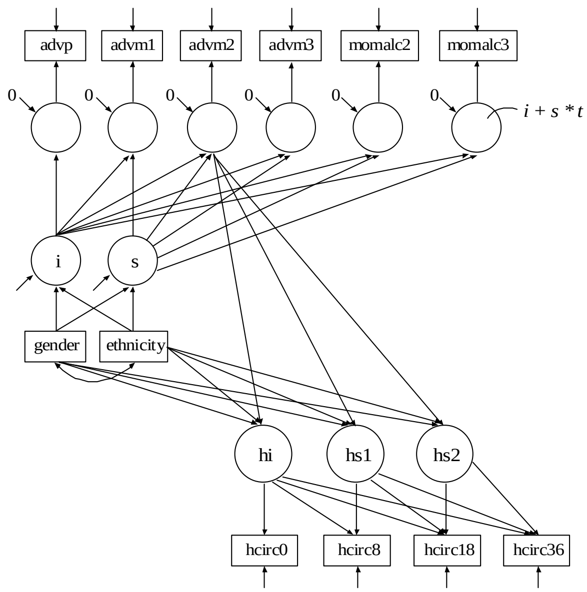
Two Linked Processes



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Input Excerpts For Two Linked Processes With Measurement Error In The Covariates

```
TITLE:     Embedded growth model with measurement error in the
           covariates and sequential processes
           advp: mother's drinking before pregnancy
           advm1-advm3: drinking in first trimester
           momalc2-momalc3: drinking in 2nd and 3rd trimesters
           hcirc0-hcirc36; head circumference
MODEL:     fadvp    BY  advp;       fadvp@0;
           fadvm1   BY  advm1;      fadvm1@0;
           fadvm2   BY  advm2;      fadvm2@0;
           fadvm3   BY  advm3;      fadvm3@0;
           fmomalc2 BY  momalc2;  fmomalc2@0;
           fmomalc3 BY  momalc3;  fmomalc3@0;
           i BY fadvp-fmomalc3@1;
           s BY fadvp@0  fadvm1@1  fadvm2*2  fadvm3*3
                  fmomalc2-fmomalc3*5 (1);
           [advp-momalc3@0 fadvp-fmomalc3@0 i s];
```

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Input Excerpts For Two Linked Processes With Measurement Error In The Covariates (Continued)

```
advp WITH advm1; advm1 WITH advm2; advm3 WITH advm2;

i s ON gender eth; s WITH i;


hi   BY  hcirc0-hcirc36@1;
hs1  BY  hcirc0@0 hcirc8@1.196 hcirc36@1.196 hcirc36@1.196;
hs2  BY  hcirc0@0 hcirc8@0 hcirc18@1 hcirc36*2;

[hcirc0-hcirc36@0 hi*34 hs1 hs2];

hs1 WITH hs2@0; hi WITH hs2@0; hi WITH hs1@0;
hi  WITH i@0; hi   WITH s@0; hs1  WITH i@0;
hi1 WITH s@0; hs2  WITH i@0; hs2  WITH s@0;

hi-hs2 ON gender eth fadvm2;
```

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Power For Growth Models

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Designing Future Studies: Power

- Computing power for growth models using Satorra-Saris (Muthén & Curran, 1997; examples)
- Computing power using Monte Carlo studies (Muthén & Muthén, 2002)
- Power calculation web site – PSMG
- Multilevel power (Miyazaki & Raudenbush, 2000; Moerbeek, Breukelen & Berger, 2000; Raudenbush, 1997; Raudenbush & Liu, 2000)
- School-based studies (Brown & Liao, 1999: Principles for designing randomized preventive trials)
- Multiple- (sequential-) cohort power
- Designs for follow-up (Brown, Indurkhia, & Kellam, 2000)

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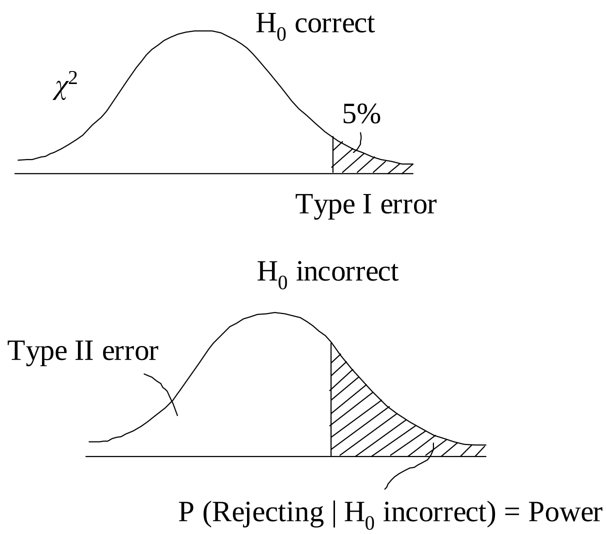
Designing Future Studies: Power



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Power Estimation For Growth Models Using Satorra & Saris (1985)

- Step 1: Create mean vector and covariance matrix for hypothesized parameter values
- Step 2: Analyze as if sample statistics and check that parameter values are recovered
- Step 3: Analyze as if sample statistics, misspecifying the model by fixing treatment effect(s) at zero
- Step 4: Use printed x^2 as an appropriate noncentrality parameter and computer power.

Muthén & Curran (1997): Artificial and real data situations.

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Input For Step 1 Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 1: Computing the population means and
           covariance matrix
DATA:      FILE IS artific.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 500;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
           i@.5;
           s@.1;
           i WITH s@0;
           y1-y4@.5;
OUTPUT:    STANDARDIZED RESIUDAL;
```

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Data For Step 1
Of Power Calculation (Continued)

```
0  0  0  0
1
0  1
0  0  1
0  0  0  1
```


Input For Step 2
Of Power Calculation

```
TITLE:      Power calculation for a growth model
            Step 2: Analyzing the population means and
            covariance matrix to check that parameters are
            recovered
DATA:       FILE IS pop.dat;
            TYPE IS MEANS COVARIANCE;
            NOBSERVATIONS = 500;
VARIABLE:   NAMES ARE y1-y4;
MODEL:      i s | y1@0 y2@1 y3@2 y4@3;
OUTPUT:     STANDARDIZED RESIUDAL;
```

Data For Step 2 Of Power Calculation (Continued)

Data From Step 1 Residual Output

0 .2 .4 .6
1
.5 1.1
.5 .7 1.4
.5 .8 1.1 1.9

237

Input For Step 3 Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 3: Analyzing the population means and
           covariance matrix with a misspecified model
DATA:      FILE IS pop.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 50;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
OUTPUT:    STANDARDIZED RESIUDAL;
```

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Step 4 Of Power Calculation

Output Excerpt From Step 3

```
Chi-Square Test of Model Fit

          Value                          9.286
          Degrees of Freedom                 6
          P-Value                        .1580
```

Power Algorithm in SAS

```
DATA POWER;
DF=1; CRIT=3.841459;
LAMBDA=9.286;
Power=(1 – (PROBCHI(CRIT, DF, LAMBDA)));
RUN;
```

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Step 4 Of Power Calculation (Continued)

Results From Power Algorithm

```
SAMPLE SIZE    POWER
    44          0.80
    50          0.85
    100         0.98
    200         0.99
```

Note: **Non-centrality parameter =**
printed chi-square value from Step 3 =
2*sample size*F

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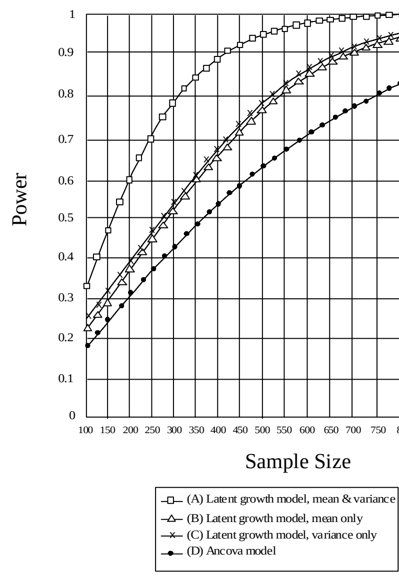

Figure 6. Power to detect a main effect of *ES* = .20 assessed at Time 5.

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MUTHÉN AND CURRAN

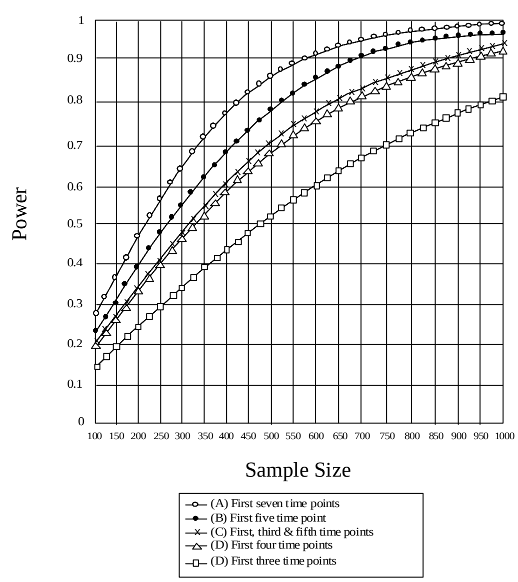

Figure 7. Power to detect a main effect of *ES* = .20 assessed at Time 5 varying as a function of total number of measurement occasions.

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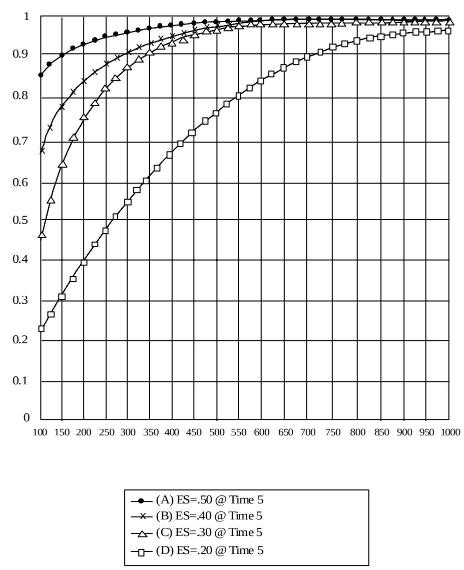

Figure 8. Power to detect various effect sizes assessed at Time 5 based on the first five measurement occasions

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Power Estimation For Growth Models Using Monte Carlo Studies

Muthén & Muthén (2002)

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Input Power Estimation For Growth Models Using Monte Carlo Studies

```
TITLE:        This is an example of a Monte Carlo
              simulation study for a linear growth model
              for a continuous outcome with missing data
              where attrition is predicted by time-
              invariant covariates (MAR)
MONTECARLO:   NAMES ARE y1-y4 x1 x2;
              NOBSERVATIONS = 500;
              NREPS = 500;
              SEED = 4533;
              CUTPOINTS = x2(1);
              MISSING = y1-y4;
```

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Input Power Estimation For Growth Models Using Monte Carlo Studies (Continued)

```
MODEL POPULATION:    x1-x2@1;
                     [x1-x2@0];
                     i s | y1@0 y2@1 y3@2 y4@3;
                     [i*1 s*2];
                     i*1; s*.2; i WITH s*.1;
                     y1-y4*.5;
                     i ON x1*1 x2*.5;
                     s ON x1*.4 x2*.25;
MODEL MISSING:       [y1-y4@-1];
                     y1 ON x1*.4 x2*.2;
                     y2 ON x1*.8 x2*.4;
                     y3 ON x1*1.6 x2*.8;
                     y4 ON x1*3.2 x2*1.6;
```

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Input Power Estimation For Growth Models Using Monte Carlo Studies (Continued)

```
ANALYSIS:     TYPE = MISSING H1;
MODEL:        i s | y1@0 y2@1 y3@2 y4@3;
              [i*1 s*2];
              i*1; s*.2; i WITH s*.1;
              y1-y4*.5;
              i ON x1*1 x2*.5;
              s ON x1*.4 x2*.25;
OUTPUT:       TECH9;
```

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Output Excerpts Power Estimation For Growth Models Using Monte Carlo Studies

Model Results

	Population	ESTIMATES Average	Std. Dev.	S.E. Average	M. S. E.	95% Cover	%Sig Coeff
I ON							
X1	1.000	1.0032	0.0598	0.0579	0.0036	0.936	1.000
X2	0.500	0.5076	0.1554	0.1570	0.0241	0.952	0.908
S ON							
X1	0.400	0.3980	0.0366	0.0349	0.0013	0.936	1.000
X2	0.250	0.2469	0.0865	0.0877	0.0075	0.938	0.830

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Cohort-Sequential Designs and Power

Considerations:

- Model identification
- Number of timepoints needed substantively
- Number of years of the study
- Number of cohorts: More gives longer timespan but greater risk of cohort differences
- Number of measurements per individual
- Number of individuals per cohort
- Number of individuals per age

Tentative conclusion:

Power most influenced by total timespan, not the number of measures per cohort

References

(To request a Muthén paper, please email bmuthen@ucla.edu.)

Analysis With Longitudinal Data

Introductory

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