Computational Issues Related To CFA

- Scale of observed variables – important to keep them on a similar scale
- Convergence – often related to starting values or the type of model being estimated
  - Program stops because maximum number of iterations has been reached
    - If no negative residual variances, either increase the number of iterations or use the preliminary parameter estimates as starting values
    - If there are large negative residual variances, try better starting values
  - Program stops before the maximum number of iterations has been reached
    - Check if variables are on a similar scale
    - Try new starting values
- Starting values – the most important parameters to give starting values to are residual variances

129

Mplus MODEL Command For CFA

MODEL command is used to describe the model to be estimated

BY statement is used to define the latent variables or factors

BY is short for “measured by”

Example 1 – standard parameterization

```
MODEL:   f1  BY  y1 y2 y3;
         f2  BY  y4 y5 y6;
```

Defaults

- Factor loading of first variable after BY is fixed to one
- Factor loadings of other variables are estimated
- Residual variances are estimated
- Residual covariances are fixed to zero
- Variances of factors are estimated
- Covariance between the exogenous factors is estimated

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Mplus MODEL Command For CFA (Continued)

Example 2 – Alternative parameterization

```
MODEL:  f1 BY y1* y2 y3;
        f2 BY y4* y5 y6;
        f1@1 f2@1;      ! or f1-f2@1;
```

131

EFA In A CFA Framework

132*

EFA In A CFA Framework

Jöreskog, K.G. (1969)

- Purpose
  - To obtain standard errors to determine if factor loadings are statistically significant
  - To obtain modification indices to determine if residual covariances are needed to represent minor factors
- Use the same number of restrictions as an exploratory factor analysis model – m^2
  - Fix factor variances to one for m restrictions
  - Fix factor loadings to zero for the remaining restrictions
    - Find an anchor item for each factor – select an item that has a large loading for the factor and small loadings for other factors
    - Fix the loading of the anchor item to zero for all of the other factors
    - Allow all other factor loadings to be free
- Will get the same model fit as EFA

133*

Selecting Anchor Items

Promax Rotated Loadings – 3 Factor Solution

	Spatial	Memory	Verbal
VISUAL	0.577	0.061	0.035
CUBES	0.602	-0.114	-0.039
PAPER	0.434	0.115	0.033
LOZENGES	**0.765**	**-0.032**	**-0.010**
GENERAL	0.152	-0.029	0.728
PARAGRAP	0.009	0.080	0.777
SENTENCE	**-0.060**	**-0.015**	**0.891**
WORDC	0.149	0.065	0.572
WORDM	0.015	0.037	0.816
WORDR	**-0.023**	**0.611**	**0.010**
NUMBERR	0.116	0.573	-0.114
OBJECT	0.127	0.678	0.043
FIGUREW	0.081	0.351	0.076

134*

Input For Holzinger-Swineford EFA In A CFA Framework Using 13 Variables

```
TITLE:      EFA in a CFA framework using 13 variables from
            Holzinger and Swineford (1939)
DATA:       FILE IS holzall.dat;
            FORMAT IS f3,2f2,f3,2f2/3x,13(1x,f3)/3x,11(1x,f3);
VARIABLE:   NAMES ARE id female grade agey agem school visual
            cubes paper lozenges general paragrap sentence wordc
            wordm addition code counting straight wordr numberr
            figurer object numberf figurew deduct numeric
            problemr series arithmet;

            USEV ARE visual cubes paper lozenges general
            paragrap sentence wordc wordm wordr numberr object
            figurew;

            USEOBS IS school EQ 0;

ANALYSIS:   ESTIMATOR = ML;
```

135*

Input For Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

```
MODEL:
         spatial BY visual-figurew*0       ! start all items at 0
                    lozenges*1             ! start anchor item at 1
                    cubes*1                ! start other large items at 1
                    sentence@0 wordr@0;    ! remove 2 indeterminacies

         memory  BY visual-figurew*0       ! start all items at 0
                    wordr*1                ! start anchor item at 1
                    object*1               ! start other large items at 1
                    lozenges@0 sentence@0; ! remove 2 indeterminacies

         verbal  BY visual-figurew*0       ! start all items at 0
                    sentence*1             ! start anchor item at 1
                    wordm*1                ! start other large items at 1
                    lozenges@0 wordr@0;    ! remove 2 indeterminacies

         spatial-verbal@1;                 ! remove 3 indeterminacies

OUTPUT:  STANDARDIZED MODINDICES(3.84) SAMPSTAT FSDETERMINACY;
```

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Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables

Tests Of Model Fit

```
Chi-Square Test of Model Fit
              Value                             39.028
              Degrees of Freedom                    42
              P-Value                           0.6022
CFI/TLI
              CFI                                1.000
              TLI                                1.009
RMSEA (Root Mean Square Error Of Approximation)
              Estimate                           0.000
              90 Percent C.I.                    0.000  0.050
              Probability RMSEA <= .05           0.949
SRMR (Standardized Root Mean Square Residual)
              Value                              0.028
```

Factor Determinacies

```
SPATIAL       0.869
MEMORY        0.841
VERBAL        0.948
```

137*

Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
SPATIAL BY					
VISUAL	**3.933**	**0.811**	**4.848**	**3.933**	**0.571**
CUBES	**2.584**	**0.559**	**4.620**	**2.584**	**0.583**
PAPER	**1.216**	**0.327**	**3.717**	**1.216**	**0.432**
LOZENGES	**6.173**	**0.765**	**8.071**	**6.173**	**0.745**
***GENERAL**	**2.278**	**1.060**	**2.149**	**2.278**	**0.196**
PARAGRAP	0.212	0.307	0.692	0.212	0.063
SENTENCE	0.000	0.000	0.000	0.000	0.000
WORDC	0.994	0.526	1.889	0.994	0.186
WORDM	0.554	0.710	0.780	0.554	0.070
WORDR	0.000	0.000	0.000	0.000	0.000
NUMBERR	0.956	1.019	0.938	0.956	0.127
OBJECT	-0.439	0.663	-0.661	-0.439	-0.096
FIGUREW	0.350	0.441	0.793	0.350	0.098

*Note that theory predicts that GENERAL loads on VERBAL only.

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Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
MEMORY BY					
VISUAL	0.580	0.808	0.718	0.580	0.084
CUBES	-0.398	0.558	-0.712	-0.398	-0.090
PAPER	0.374	0.333	1.123	0.374	0.133
LOZENGES	0.000	0.000	0.000	0.000	0.000
GENERAL	-0.100	1.103	-0.091	-0.100	-0.009
PARAGRAP	0.318	0.309	1.030	0.318	0.094
SENTENCE	0.000	0.000	0.000	0.000	0.000
WORDC	0.436	0.540	0.808	0.436	0.082
WORDM	0.425	0.720	0.590	0.425	0.054
WORDR	**6.541**	**1.058**	**6.180**	**6.541**	**0.606**
NUMBERR	**4.291**	**0.977**	**4.392**	**4.291**	**0.571**
OBJECT	**3.040**	**0.646**	**4.704**	**3.040**	**0.668**
FIGUREW	**1.264**	**0.433**	**2.923**	**1.264**	**0.353**

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Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
VERBAL BY					
VISUAL	0.265	0.811	0.326	0.265	0.038
CUBES	-0.129	0.546	-0.236	-0.129	-0.029
PAPER	0.096	0.327	0.294	0.096	0.034
LOZENGES	0.000	0.000	0.000	0.00	0.000
GENERAL	**8.130**	**1.058**	**7.682**	**8.130**	**0.700**
PARAGRAP	**2.501**	**0.303**	**8.264**	**2.501**	**0.744**
SENTENCE	**3.954**	**0.322**	**12.263**	**3.954**	**0.853**
WORDC	**2.927**	**0.517**	**5.656**	**2.927**	**0.548**
WORDM	**6.191**	**0.707**	**8.751**	**6.191**	**0.782**
WORDR	0.000	0.000	0.000	0.000	0.000
NUMBERR	-0.870	1.033	-0.842	-0.870	-0.116
OBJECT	0.139	0.653	0.212	0.139	0.030
FIGUREW	0.247	0.433	0.570	0.247	0.069

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Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
VERBAL WITH					
SPATIAL	0.467	0.119	3.937	0.467	0.467
MEMORY WITH					
SPATIAL	0.371	0.171	2.173	0.371	0.371
VERBAL	0.459	0.144	3.181	0.459	0.459
Variances					
SPATIAL	1.000	0.000	0.000	1.000	1.000
VERBAL	1.000	0.000	0.000	1.000	1.000
MEMORY	1.000	0.000	0.000	1.000	1.000

141*

Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Residual Variances					
VISUAL	28.758	4.325	6.649	28.758	0.606
CUBES	13.795	2.049	6.732	13.795	0.703
PAPER	5.801	0.761	7.619	5.801	0.734
LOZENGES	30.640	7.063	4.338	30.640	0.446
GENERAL	47.239	6.824	6.923	47.239	0.350
PARAGRAP	3.637	0.544	6.684	3.637	0.321
SENTENCE	5.831	1.042	5.598	5.831	0.272
WORDC	14.547	1.864	7.803	14.547	0.510
WORDM	18.122	2.878	6.298	18.122	0.289
WORDR	73.589	12.422	5.924	73.589	0.632
NUMBERR	37.595	5.998	6.268	37.595	0.665
OBJECT	11.939	2.377	5.022	11.939	0.576
FIGUREW	10.368	1.319	7.860	10.368	0.807

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Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

R-Square

```
Observed           R-Square    ! R-Square =
Variable                       ! 1 – STDYX (residual) = Reliability
                               ! when no covariates are in the model
VISUAL                0.394
CUBES                 0.297
PAPER                 0.266
LOZENGES              0.554
GENERAL               0.650
PARAGRAP              0.679
SENTENCE              0.728
WORDC                 0.490
WORDM                 0.711
WORDR                 0.368
NUMBERR               0.335
OBJECT                0.424
FIGUREW               0.193
```

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Output Excerpts Holzinger-Swineford EFA In A CFA Framework Using 13 Variables (Continued)

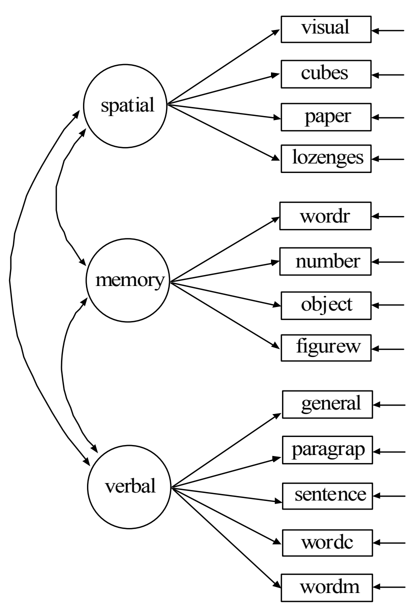
Model Modification Indices

```
                        M.I.   E.P.C.  Std E.P.C.    StdYX E.P.C.

WITH Statements

WORDC    WITH SENTENCE  6.586    2.657      2.657     0.107
WORDM    WITH GENERAL   7.121    9.555      9.555     0.104
WORDM    WITH SENTENCE  6.557   -4.238     -4.238    -0.116
```

144*

Simple Structure CFA

145



146

Input Excerpts For Holzinger-Swineford Simple Structure CFA Using 13 Variables

```
MODEL:      spatial BY visual-lozenges;
            memory BY wordr-figurew;
            verbal BY general-wordm;

OUTPUT:     STANDARDIZED MODINDICES(3.84) SAMPSTAT FSDETERMINACY;
```

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Output Excerpts Holzinger-Swineford Simple Structure CFA Using 13 Variables

Tests Of Model Fit

```
Chi-Square Test of Model Fit
             Value                                 56.254
             Degrees of Freedom                        62
             P-Value                               0.6817
CFI/TLI
             CFI                                    1.000
             TLI                                    1.012
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                               0.000
             90 Percent C.I.                        0.000  0.041
             Probability RMSEA <= .05               0.983
SRMR (Standardized Root Mean Square Residual)
             Value                                  0.046
```

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Output Excerpts Holzinger-Swineford Simple Structure CFA Using 13 Variables (Continued)

Note: Model fit is better than with the EFA in a CFA framework (p= .6022). This is because the parameters that were fixed to zero were not significant. Thus the gain in degrees of freedom resulted in a higher p-value.

The chi-square difference test between the EFA in a CFA framework and the Simple Structure CFA models is not significant: Chi-square value of 17.23 with 20 degrees of freedom.

Factor Determinacies

```
SPATIAL       0.867
MEMORY        0.835
VERBAL        0.954
```

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Output Excerpts Holzinger-Swineford Simple Structure CFA Using 13 Variables (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
SPATIAL BY					
VISUAL	1.000	.000	.000	4.539	.659
CUBES	.481	.102	4.691	2.182	.492
PAPER	.329	.066	4.975	1.491	.530
LOZENGES	1.303	.219	5.941	5.915	.714
MEMORY BY					
WORDR	1.000	.000	.000	6.527	.605
NUMBERR	.642	.142	4.534	4.191	.557
OBJECT	.435	.091	4.776	2.840	.624
FIGUREW	.247	.063	3.937	1.613	.450
VERBAL BY					
GENERAL	1.000	.000	.000	9.363	.806
PARAGRAP	.295	.027	11.077	2.766	.822
SENTENCE	.413	.037	11.294	3.866	.834
WORDC	.394	.044	8.857	3.688	.691
WORDM	.716	.062	11.513	6.707	.847

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Output Excerpts Holzinger-Swineford Simple Structure CFA Using 13 Variables (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
VERBAL WITH					
SPATIAL	25.118	5.700	4.407	.591	.591
MEMORY WITH					
SPATIAL	13.323	4.329	3.077	.450	.450
VERBAL	31.883	8.340	3.823	.522	.522
Variances					
SPATIAL	20.597	5.450	3.779	1.000	1.000
VERBAL	87.646	15.363	5.705	1.000	1.000
MEMORY	42.606	13.205	3.226	1.000	1.000

Output Excerpts Holzinger-Swineford Simple Structure CFA Using 13 Variables (Continued)

R-Square

VISUAL	0.434
CUBES	0.243
PAPER	0.281
LOZENGES	0.509
GENERAL	0.650
PARAGRAP	0.676
SENTENCE	0.696
WORDC	0.477
WORDM	0.717
WORDR	0.366
NUMBERR	0.311
OBJECT	0.389
FIGUREW	0.203

Output Excerpts Holzinger-Swineford Simple Structure CFA Using 13 Variables (Continued)

Model Modification Indices

```
                             M.I.    E.P.C.  Std E.P.C.   StdYX E.P.C.
WITH Statements

PARAGRAP WITH GENERAL   4.170    -3.108     -3.108       -0.080
WORDC    WITH SENTENCE  4.586     2.207      2.207        0.089
WORDM    WITH GENERAL   4.552     7.582      7.582        0.082
```


Special Factor Analysis Models

Bi-Factor Model

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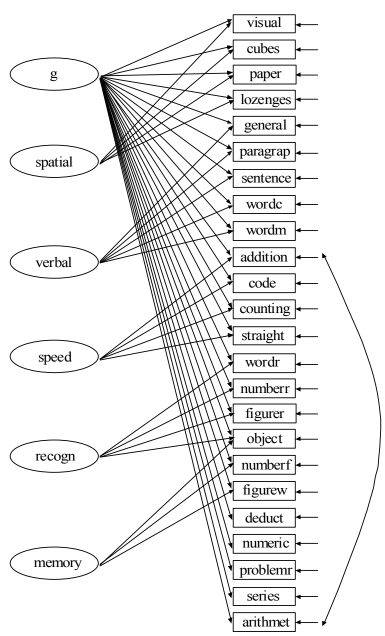

156

Input Excerpts Holzinger-Swineford General-Specific (Bi-Factor) Factor Model

```
MODEL:      g BY visual-arithmet;
            spatial BY visual-lozenges;
            verbal BY general-wordm;
            speed BY addition-straight;
            recogn BY wordr-object;
            memory BY numberf object figurew;

!           uncorrelated factors because of the general factor:

            g WITH spatial-memory @0;
            spatial WITH verbal-memory @0;
            verbal WITH speed-memory @0;
            speed WITH recogn-memory @0;
            recogn WITH memory @0;

!           correlated residual (“doublet factor”):

            addition WITH arithmet;

OUTPUT:     STANDARDIZED MODINDICES(3.84) SAMPSTAT FSDTERMINACY;
```

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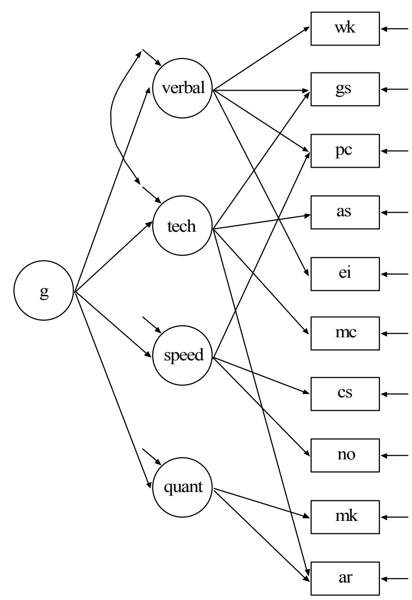
Second-Order Factor Model

158



Input For Second-Order Factor Analysis Model

```
TITLE:      Second-order factor analysis model
DATA:       FILE IS asvab.dat;
            !  Armed services vocational aptitude battery
            NOBSERVATIONS = 20422;
            TYPE=COVARIANCE;
VARIABLE:   NAMES ARE ar wk pc mk gs no cs as mc ei;
            USEV = wk gs pc as ei mc cs no mk ar;

            !WK   Word Knowledge
            !GS   General Science
            !PC   Paragraph Comprehension
            !AS   Auto and Shop Information
            !EI   Electronics information
            !MC   Mechanical Comprehension
            !CS   Coding Speed
            !NO   Numerical Operations
            !MK   Mathematical Knowledge
            !AR   Arithmetic Reasoning

ANALYSIS:   ESTIMATOR = ML;
```

Input For Second-Order Factor Analysis Model (Continued)

```
MODEL:     verbal BY wk gs pc ei;
           tech BY gs mc ar;
           speed BY pc cs no;
           quant BY mk ar;
           g BY verbal tech speed quant;
           tech WITH verbal;

OUTPUT:    SAMPSTAT MOD(0) STAND TECH1 RESIDUAL;
```

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Further Readings On CFA

Bollen, K.A. (1989). Structural equations with latent variables. New York: John Wiley.

Joreskog, K.G. (1969). A general approach to confirmatory maximum likelihood factor analysis. Psychometrika, 34, 183-202.

Lawley, D.N. & Maxwell, A.E. (1971). Factor analysis as a statistical method. London: Butterworths.

Long, S. (1983). Confirmatory factor analysis. Sage University Paper series on Quantitative Applications in the Social Sciences, No 33. Beverly Hills, CA: Sage.

Mulaik, S. (1972). The foundations of factor analysis. McGraw-Hill.

162

Measurement Invariance And Population Heterogeneity

163

Models To Study Measurement Invariance And Population Heterogeneity

To further study a set of factors or latent variables established by an EFA/CFA, questions can be asked about the invariance of the measures and the heterogeneity of populations.

Measurement Invariance – Does the factor model hold in other populations or at other time points?

- Same number of factors
- Zero loadings in the same positions
- Equality of factor loadings
- Equality of intercepts
  - Test difficulty

Population Heterogeneity – Are the factor means, variances, and covariances the same for different populations?

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Models To Study Measurement Invariance And Population Heterogeneity (Continued)

Models To Study Measurement Invariance and Population Heterogeneity

- CFA with covariates
  - Parsimonious
  - Small sample advantage
  - Advantageous with many groups
- Multiple group analysis
  - More parameters to represent non-invariance
    - Factor loadings and observed residual variances/covariances in addition to intercepts
    - Factor variances and covariances in addition to means
  - Interactions

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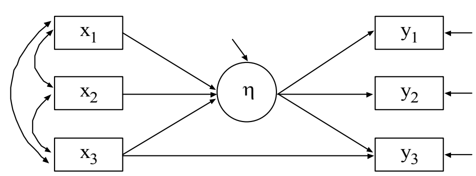
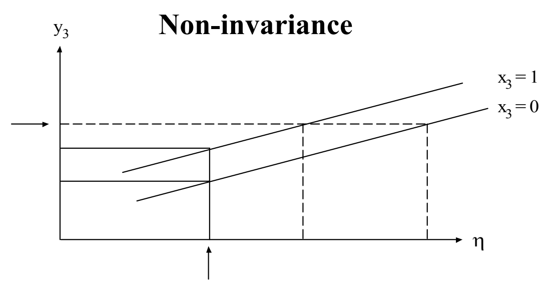
CFA With Covariates

Non-invariance



Non-invariance



Conditional on η, y is different for the two groups

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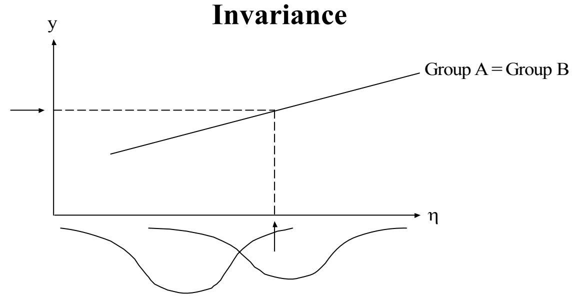
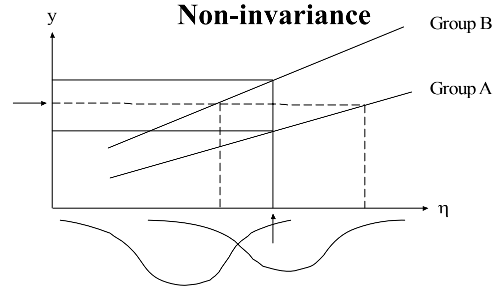
Multiple Group Analysis

Invariance



Non-invariance



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CFA With Covariates (MIMIC)

168

CFA With Covariates (MIMIC)

Used to study the effects of covariates or background variables on the factors and outcome variables to understand measurement invariance and heterogeneity

- Measurement non-invariance – direct relationships between the covariates and factor indicators that are not mediated by the factors – if they are significant, this indicates measurement non-invariance due to differential item functioning (DIF)

- Population Heterogeneity – relationships between the covariates and the factors – if they are significant, this indicates that the factor means are different for different levels of the covariates.

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CFA With Covariates (MIMIC) (Continued)

Model Assumptions

- Same factor loadings and observed residual variances / covariances for all levels of the covariates
- Same factor variances and covariances for all levels of the covariates

Model identification, estimation, testing, and modification are the same as for CFA.

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Steps In CFA With Covariates

- Establish a CFA or EFA/CFA model
- Add covariates – check that factor structure does not change and study modification indices for possible direct effects
- Add direct effects suggested by modification indices – check that factor structure does not change
- Interpret the model
  - Factors
  - Effects of covariates on factors
  - Direct effects of covariates on factor indicators

171

NELS Data

The NELS data consist of 16 testlets developed to measure the achievement areas of reading, math, science, and other school subjects. The sample consists of 4,154 eighth graders from urban, public schools.

Data for the analysis include five reading testlets and four math testlets. The entire sample is used.

Variables

rlit – reading literature
rsci – reading science
rpoet – reading poetry
rbiog – reading biography
rhist – reading history

malg – math algebra
marith – math arithmetic
mgeom – math geometry
mprob – math probability

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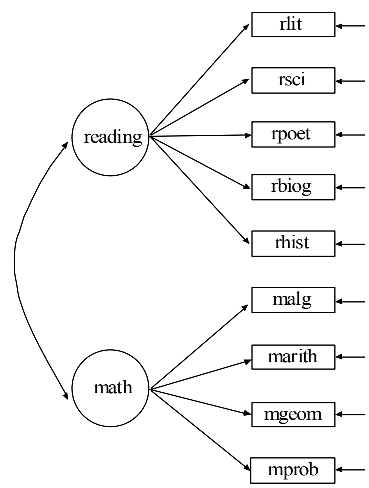

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Input For NELS CFA

```
TITLE:      CFA using NELS data
DATA:       FILE IS ft21.dat;
VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit hhist gender schoolid minorc;

            USEVARIABLES ARE rlit-mprob;

MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;

OUTPUT:     STANDARDIZED MODINDICES;
```

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Output Excerpts NELS CFA

Tests Of Model Fit

```
Chi-Square Test of Model Fit
             Value                              128.872
             Degrees of Freedom                      26
             P-Value                             0.0000
CFI/TLI
             CFI                                  0.993
             TLI                                  0.990
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                             0.031
             90 Percent C.I.                      0.026  0.036
             Probability RMSEA <= .05             1.000
SRMR (Standardized Root Mean Square Residual)
             Value                                0.016
```

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Output Excerpts NELS CFA (Continued)

Model Results

```
                  Estimates      S.E.  Est./S.E.    Std     StdYX

 READING   BY
    RLIT            1.000        .000      .000     .845     .657
    RSCI            1.383        .038    36.451    1.168     .672
    RPOET           1.130        .030    37.558     .955     .698
    RBIOG           1.300        .034    37.791    1.098     .703
    RHIST           1.287        .037    34.436    1.087     .627
 MATH      BY
    MALG            1.000        .000      .000    1.018     .868
    MARITH          1.026        .015    69.297    1.045     .890
    MGEOM            .655        .020    32.637     .667     .494
    MPROB           1.066        .028    38.300    1.086     .565
 MATH      WITH
    READING          .723        .024    30.067     .840     .840
```

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Output Excerpts NELS CFA (Continued)

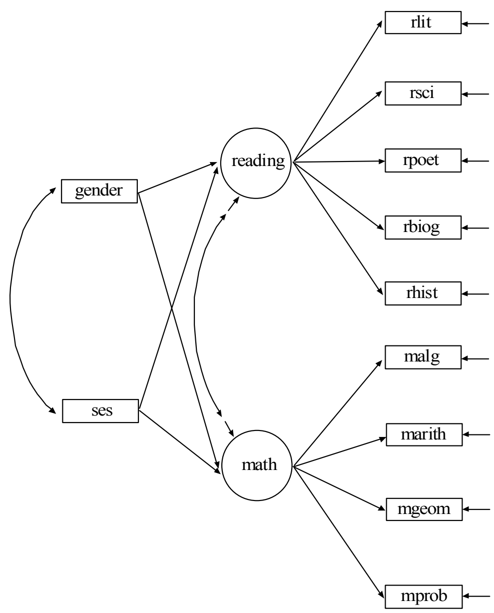
Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Residual Variances					
RLIT	.939	.024	39.516	.939	.568
RSCI	1.657	.042	39.000	1.657	.548
RPOET	.962	.025	37.986	.962	.513
RBIOG	1.234	.033	37.745	1.234	.506
RHIST	1.822	.045	40.416	1.822	.606
MALG	.339	.012	27.759	.339	.246
MARITH	.285	.012	24.067	.285	.207
MGEOM	1.379	.031	43.922	1.379	.756
MPROB	2.518	.058	43.165	2.518	.681
Variances					
READING	.714	.032	22.231	1.000	1.000
MATH	1.037	.031	33.659	1.000	1.000

Output Excerpts NELS CFA (Continued)

R-Square

RLIT	.432
RSCI	.452
RPOET	.487
RBIOG	.494
RHIST	.394
MALG	.754
MARITH	.793
MGEOM	.244
MPROB	.319



Input For NELS CFA With Covariates

```
TITLE:      CFA with covariates using NELS data

DATA:       FILE IS ft21.dat;

VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit hhist gender schoolid minorc;

            USEVARIABLES ARE rlit-mprob ses gender;

MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;

            reading math ON ses gender;   ! female = 0, male = 1

OUTPUT:     STANDARDIZED MODINDICES (3.84);
```

Output Excerpts NELS CFA With Covariates

Tests Of Model Fit

```
Chi-Square Test of Model Fit
             Value                          202.935
             Degrees of Freedom                  40
             P-Value                         0.0000
CFI/TLI
             CFI                              0.990
             TLI                              0.986
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                         0.031
             90 Percent C.I.                  0.027  0.036
             Probability RMSEA <= .05         1.000
SRMR (Standardized Root Mean Square Residual)
             Value                            0.018
```


Output Excerpts NELS CFA With Covariates

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
READING BY					
RLIT	1.000	.000	.000	.846	.658
RSCI	1.370	.038	36.437	1.159	.667
RPOET	1.133	.030	37.907	.959	.700
RBIOG	1.296	.034	37.998	1.097	.702
RHIST	1.291	.037	34.758	1.092	.630
MATH BY					
MALG	1.000	.000	.000	1.015	.866
MARITH	1.031	.015	70.136	1.047	.892
MGEOM	.659	.020	32.794	.669	.495
MPROB	1.071	.028	38.435	1.088	.566

Output Excerpts NELS CFA With Covariates (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
READING ON					
SES	**.344**	**.014**	**24.858**	**.407**	**.438**
GENDER	**-.186**	**.027**	**-6.901**	**-.220**	**-.110**
MATH ON					
SES	**.418**	**.015**	**28.790**	**.412**	**.444**
GENDER	**.044**	**.030**	**1.457**	**.044**	**022**
MATH WITH					
READING	.558	.019	29.142	.649	.649

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Output Excerpts NELS CFA With Covariates (Continued)

Residual Variances

RLIT	.937	.024	39.695	.937	.567
RSCI	1.679	.043	39.407	1.679	.555
RPOET	.955	.025	38.136	.955	.510
RBIOG	1.237	.033	38.046	1.237	.507
RHIST	1.812	.045	40.521	1.812	.603
MALG	.345	.012	28.752	.345	.251
MARITH	.281	.012	24.388	.281	.204
MGEOM	1.377	.031	43.946	1.377	.754
MPROB	2.513	.058	43.207	2.513	.680
READING	.572	.026	21.920	.799	.799
MATH	.826	.025	32.943	.801	.801

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Output Excerpts NELS CFA With Covariates (Continued)

R-Square

RLIT	.433
RSCI	.445
RPOET	.490
RBIOG	.493
RHIST	.397
MALG	.749
MARITH	.796
MGEOM	.246
MPROB	.320

Latent Variable	R-Square
READING	.201
MATH	.199

Input For Modification Indices For Direct Effects NELS CFA With Covariates

```
TITLE:      Modification indices for direct effects
            CFA with covariates using NELS data

DATA:       FILE IS ft21.dat;

VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit hhist gender schoolid minorc;

            USEVARIABLES ARE rlit-mprob ses gender;

MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;

            reading math ON ses gender;   !female = 0, male = 1

            rlit-mprob ON ses-gender@0;

OUTPUT:     STANDARDIZED MODINDICES(3.84);
```

Output Excerpts Modification Indices For Direct Effects NELS CFA With Covariates

Modification Indices

		M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
RSCI	**ON GENDER**	**31.730**	**0.253**	**0.253**	**0.073**
RPOET	ON GENDER	12.715	-0.124	-0.124	-0.045
RHIST	ON SES	6.579	0.062	0.062	0.038
MALG	**ON GENDER**	**26.616**	**-0.120**	**-0.120**	**-0.051**
MARITH	ON GENDER	10.083	0.075	0.075	0.032
MGEON	ON SES	4.201	0.040	0.040	0.032
MPROB	ON GENDER	7.922	0.143	0.143	0.037

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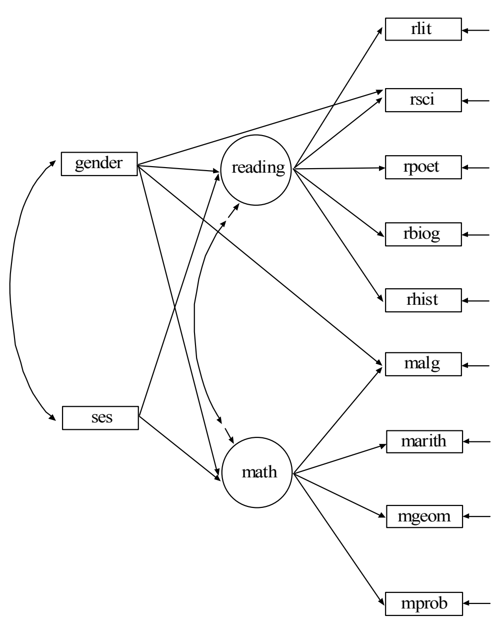

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Summary Of Analysis Results For NELS CFA With Covariates And Direct Effects

Model	Chi-square (d.f.)	Difference (d.f. diff.)
No direct effects	202.935 (40)	
rsci ON gender	171.006 (39)	31.929* (1)
rsci ON gender and malg ON gender	144.728 (38)	26.728* (1)

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Input For NELS CFA With Covariates And Two Direct Effects

```
TITLE:      CFA with covariates and two direct effects using
            NELS data

DATA:       FILE IS ft21.dat;

VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit hhist gender schoolid minorc;

            USEVARIABLES ARE rlit-mprob ses gender;

MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;

            reading math ON  ses gender;   !female = 0, male = 1

            rsci ON gender;
            malg ON gender;

OUTPUT:     STANDARDIZED MODINDICES(3.84);
```

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Output Excerpts NELS CFA With Covariates And Two Direct Effects

Tests Of Model Fit

```
Chi-Square Test of Model Fit
             Value                              144.278
             Degrees of Freedom                      38
             P-Value                             0.0000
CFI/TLI
             CFI                                  0.993
             TLI                                  0.991
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                             0.026
             90 Percent C.I.                      0.022  0.031
             Probability RMSEA <= .05             1.000
SRMR (Standardized Root Mean Square Residual)
             Value                                0.014
```

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Output Excerpts NELS CFA With Covariates And Two Direct Effects (Continued)

Model Results

```
                  Estimates       S.E.  Est./S.E.    Std     StdYX
 READING   BY
    RLIT             1.000        .000      .000     .846     .658
    RSCI             1.389        .038    36.609    1.175     .676
    RPOET            1.133        .030    37.958     .959     .701
    RBIOG            1.294        .034    37.991    1.095     .701
    RHIST            1.290        .037    34.760    1.091     .630
 MATH       BY
    MALG             1.000        .000      .000    1.019     .869
    MARITH           1.027        .015    70.300    1.047     .892
    MGEOM             .657        .020    32.833     .670     .496
    MPROB            1.068        .028    38.524    1.089     .566
```

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Output Excerpts NELS CFA With Covariates And Two Direct Effects (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
READING ON					
SES	.343	.014	24.854	.406	.437
GENDER	-.222	.028	-7.983	-.262	-.131
MATH ON					
SES	.419	.015	28.807	.411	.444
GENDER	**.092**	**.032**	**2.873**	**.090**	**.045**
RSCI ON					
GENDER	**.254**	**.045**	**5.649**	**.254**	**.073**
MALG ON					
GENDER	**-.121**	**.023**	**-5.171**	**-.121**	**-.051**

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Interpretation Of Direct Effects

Rsci On Gender

- Indirect effect of gender on rsci
  - Reading factor has a negative relationship with gender – males have a lower mean than females on the reading factor
  - Rsci has a positive loading on the reading factor
  - Conclusion: Males are expected to have a lower mean on rsci

- Direct effect of gender on rsci
  - Direct effect is positive – for a given reading factor value, males do better than expected on rsci
  - Conclusion – rsci is not invariant. Males may have had more exposure to science reading.

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Interpretation Of Direct Effects (Continued)

Malg On Gender

- Indirect effect of gender on malg
  - Math factor has a positive relationship with gender – males have a higher mean than females in math
  - Malg has a positive loading on the math factor
  - Conclusion: Males are expected to have a higher mean on malg
- Direct effect of gender on malg
  - Direct effect is negative – for a given math factor value, males do worse than expected on malg
  - Conclusion: malg is not invariant

Multiple Group Analysis

Multiple Group Analysis

Used to study group differences in measurement and structural parameters by simultaneous analysis of several groups of individuals

Advantages Of Multiple Group Analysis Versus Factor Analysis With Covariates

- More parameters to represent non-invariance
  - Factor loadings and observed residual variances/covariances in addition to intercepts
  - Factor variances and covariances in addition to means
- Interactions

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Multiple Group Analysis (Continued)

Disadvantages Of Multiple Group Analysis Versus Factor Analysis With Covariates

- Less parsimonious model
- Requires sufficiently large sample size for each group
- Difficult to carry out with many groups

Model Specification

- Comparison of factor variances and covariances meaningful only when factor loadings are invariant
- Comparison of factor means meaningful only when factor loadings and measurement intercepts are invariant
- Partial invariance possible

Model identification, estimation, testing, and modification are the same as for CFA.

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Steps In Multiple Group Analysis

- Fit the model separately in each group
- Fit the model in all groups allowing all parameters to be free
- Fit the model in all groups holding factor loadings equal to test the invariance of the factor loadings
- Fit the model in all groups holding factor loadings and intercepts equal to test the invariance of the intercepts
- Add covariates
- Modify the model

Mplus Input For Multiple Group Analysis

- General rules
  - MODEL command is used to describe the overall analysis model for all groups
  - Group-specific MODEL commands are used to specify differences between the overall analysis model and the model for that group
  - Equalities specified in the MODEL command apply across groups
  - Equalities specified in the group-specific MODEL commands apply to only the specific group

Mplus Input For Multiple Group Analysis (Continued)

- Defaults
  - Factor loadings are held equal across the groups
  - All other free parameters are not held equal across groups
  - When means are included in the model
    - Intercepts of observed variables are held equal across group
    - Factor means are fixed at zero in the first group and are free to be estimated in the other groups

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Mplus Input For Multiple Group Analysis (Continued)

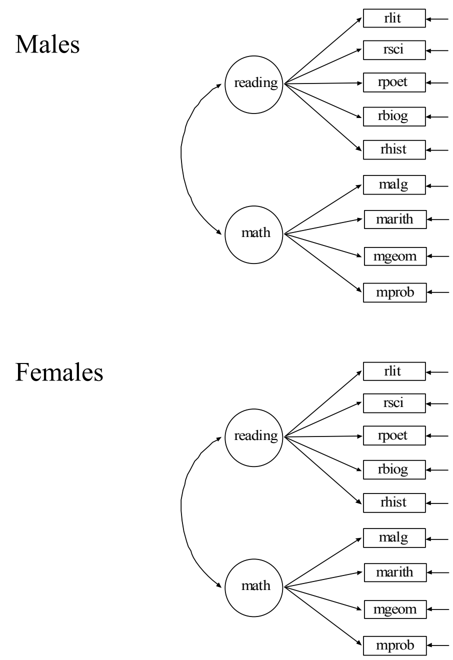
- Example 1 – factor loading invariance across groups

```
MODEL:      f1 BY y1 y2 y3;
            f2 BY y4 y5 y6;
```

- Example 2 – factor loading non-invariance for 2 groups

```
MODEL:      f1 BY y1 y2 y3;
            f2 BY y4 y5 y6;
MODEL g2:   f1 BY y2 y3;
            f2 BY y5 y6;
```

202



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Inputs For NELS Single Group Analyses Without Measurement Invariance

Single Group Analyses

```
TITLE:      Single group CFA for males using NELS data

DATA:       FILE IS ft21.dat;

VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit gender schoolid minorc;

            USEVARIABLES ARE rlit-mprob;

            USEOBSERVATIONS ARE (gender EQ 1);  !  change 1 to
                                                !  0 for females
MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;
```

204

Input For NELS Multiple Group Analysis Without Measurement Invariance

```
TITLE:        Multiple group CFA for males and females using NELS
              data with no measurement invariance

DATA:         FILE IS ft21.dat;

VARIABLE:     NAMES ARE ses rlit rsci rpoet rbiog rhist malg
              marith mgeom mprob searth schem slife smeth hgeog
              hcit gender schoolid minorc;

              GROUPING IS gender (0=female 1=male);

              USEVARIABLES ARE rlit-mprob;

MODEL:        reading BY rlit-rhist;
              math BY malg-mprob;

MODEL male:   reading BY rsci-rhist;
              math BY marith-mprob;
```

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Summary Of Analysis Results For NELS Single And Multiple Group Analyses Without Measurement Invariance

	Chi-square	RMSEA
Males (n=2048)	72.555 (26) .0000	.030
Females (n=2106)	86.274 (26) .0000	.033
Together (n=4154)	158.829 (52) .0000	.031

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Input For NELS Multiple Group Analyses With Measurement Invariance

Invariance Of Factor Loadings

```
TITLE:      Multiple group CFA for males and females using NELS
            data with measurement invariance of factor loadings

DATA:       FILE IS ft21.dat;

VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit gender schoolid minorc;
            GROUPING IS gender (0=female 1=male);
            USEVARIABLES ARE rlit-mprob;

ANALYSIS:   MODEL = NOMEANSTRUCTURE;

MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;

OUTPUT:     STANDARDIZED MODINDICES(3.84);
```

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Input For NELS Multiple Group Analyses With Measurement Invariance (Continued)

Invariance Of Factor Loadings And Intercepts

```
TITLE:      Multiple group CFA for males and females using NELS
            data with measurement invariance of factor loadings
            and intercepts

DATA:       FILE IS ft21.dat;

VARIABLE:   NAMES ARE ses rlit rsci rpoet rbiog rhist malg
            marith mgeom mprob searth schem slife smeth hgeog
            hcit gender schoolid minorc;
            GROUPING IS gender (0=female 1=male);
            USEVARIABLES ARE rlit-mprob;

MODEL:      reading BY rlit-rhist;
            math BY malg-mprob;

OUTPUT:     STANDARDIZED MODINDICES(3.84);
```

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Summary Of Analysis Results For NELS Single And Multiple Group Analyses With Measurement Invariance

Model	Chi-square	Difference
Measurement non-invariance	158.829 (52)	
Factor loading invariance	170.386 (59)	11.557 (7)
Factor loading and intercept invariance	238.847 (66)	68.461* (7)

Summary Of Analysis Results For NELS Single And Multiple Group Analyses With Measurement Invariance (Continued)

Explanation of Chi-Square Differences

Factor loading invariance	(7)	7 factor loadings instead of 14
Factor loading and intercept invariance	(7)	9 intercepts and 2 factor means instead of 18 intercepts

Summary Of Analysis Results For NELS Single And Multiple Group Analyses With Measurement Invariance (Continued)

Modification Indices (Excerpts)

```
Group MALE
                         M.I.        E.P.C.    Std       StdYX
                                               E.P.C.    E.P.C.

Means/Intercepts/Thresholds

[ RSCI    ]            31.794        .154      .154      .089
[ RPOET   ]            12.856       -.081     -.081     -.058
[ MALG    ]            26.574       -.085     -.085     -.071
[ MARITH  ]            10.084        .056      .056      .047
[ MPROB   ]             7.903        .075      .075      .039
```

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Input Excerpts For NELS Multiple Group Analysis With Partial Measurement Invariance

```
MODEL:        reading BY rlit-rhist;
              math BY malg-mprob;

MODEL male:   [rsci malg];

OUTPUT:       STANDARDIZED  MODINDICES (3.84);
```

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Summary Of Analysis Results For NELS Multiple Group Analysis With Partial Measurement Invariance

Model	Chi-square	Difference
Measurement non-invariance	170.386 (59)	
Factor loading and partial intercept invariance	180.110 (64)	9.724 (5)

Input Excerpts For NELS Multiple Group Analysis With Partial Measurement Invariance And Invariant Residual Variances

```
MODEL:       reading    BY  rlit-rhist;
             math       BY  malg-mprob;
             rlit-mprob (1-9);

MODEL male:  [rsci malg];

OUTPUT:      STANDARDIZED  MODINDICES (3.84);
```

Summary Of Analysis Results For NELS Multiple Group Analysis With Partial Measurement Invariance And Invariant Residual Variances

Model	Chi-square	Difference
Partial invariance	180.110 (64)	
Item residual invariance	197.513 (73)	17.403 (9)*

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Input Excerpts For NELS Multiple Group Analysis With Partial Measurement Invariance And Invariant Factor Variances And Covariance: A Test Of Population Heterogeneity

```
MODEL:        reading BY rlit-rhist;
              math BY malg-mprob;

              reading (1);
              math    (2);
              reading WITH math (3);

MODEL male:   [rsci malg];

OUTPUT:       STANDARDIZED MODINDICES (3.84);
```

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Summary Of Analysis Results For NELS Multiple Group Analysis With Partial Measurement Invariance And Invariant Factor Variances And Covariance: A Test Of Population Heterogeneity

Model	Chi-square	Difference
Partial invariance	180.110 (64)	
Invariant factor variances and covariance	183.442 (67)	3.312 (3)

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Input Excerpts For NELS Multiple Group Analysis With Partial Measurement Invariance And Invariant Factor Variances, Covariance, And Means: A Test Of Population Heterogeneity

```
MODEL:        reading BY rlit-rhist;
              math BY malg-mprob;
              reading (1);
              math    (2);
              reading WITH math (3);

MODEL male :  [rsci malg reading@0 math@0];

OUTPUT:       STANDARDIZED MODINDICES (3.84);
```

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Summary Of Analysis Results For NELS Multiple Group Analysis With Partial Measurement Invariance And Invariant Factor Variances, Covariance, And Means: A Test Of Population Heterogeneity

Model	Chi-square	Difference
Partial invariance	180.110 (64)	
Invariant factor variances and covariance	183.422 (67)	3.312 (3)
Invariant factor variances, covariance, and means	340.498 (69)	157.076 (2)*

219

Technical Aspects Of Multiple-Group Factor Analysis Modeling

$$\boldsymbol{y}_{ig} = \boldsymbol{\nu}_g + \boldsymbol{\Lambda}_g\, \boldsymbol{\eta}_{ig} + \boldsymbol{\varepsilon}_{ig}\,, \qquad (21)$$

$$E(\boldsymbol{y}_g) = \boldsymbol{\nu}_g + \boldsymbol{\Lambda}_g\, \boldsymbol{\alpha}_g\,, \qquad (22)$$

$$V(\boldsymbol{y}_g) = \boldsymbol{\Lambda}_g\, \boldsymbol{\Psi}_g\, \boldsymbol{\Lambda}_g' + \boldsymbol{\Theta}_g\,. \qquad (23)$$

ML estimation with G *independently* observed groups:

$$F_{ML}\,(\boldsymbol{\pi}) = 1/2 \sum_{g=1}^{G} \{n_g[\ln|\boldsymbol{\Sigma}_g| + trace\,(\boldsymbol{\Sigma}_g^{-1}\boldsymbol{T}_g) - \ln|\boldsymbol{S}_g| - p]\}/n, \qquad (24)$$

where n_g is the sample size in group g, $n = \Sigma_g^G n_g$, and

$$\boldsymbol{T}_g = \boldsymbol{S}_g + (\bar{\boldsymbol{y}}_g - \boldsymbol{\mu}_g)(\bar{\boldsymbol{y}}_g - \boldsymbol{\mu}_g)' \qquad (25)$$

(e.g. Jöreskog & Sörbom, 1979; Browne & Arminger, 1995).

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Technical Aspects Of Multiple-Group Factor Analysis Modeling (Continued)

Two main cases:

- No mean structure
  - Assume $\boldsymbol{\Lambda}$ invariance
  - Study ($\boldsymbol{\Theta}_g$ and) $\boldsymbol{\Psi}_g$ differences
  - ($\boldsymbol{\nu}_g$ free, $\boldsymbol{\alpha} = \mathbf{0}$, so that $\widehat{\boldsymbol{\mu}}_g = \bar{\boldsymbol{y}}_g$)

- Mean structure
  - Assume $\boldsymbol{\nu}$ and $\boldsymbol{\Lambda}$ invariance
  - Study ($\boldsymbol{\Theta}_g$ and) $\boldsymbol{\alpha}_g$ and $\boldsymbol{\Psi}_g$ differences ($\boldsymbol{\alpha}_1 = \mathbf{0}$)

221

Further Readings On MIMIC And Multiple-Group Analysis

Joreskog, K.G. (1971). Simultaneous factor analysis in several populations. Psychometrika, 36, 409-426.

Meredith, W. (1964). Notes on factorial invariance. Psychometrika, 29, 177-185.

Meredith, W. (1993). Measurement invariance, factor analysis and factorial invariance. Psychometrika, 58, 525-543.

Muthen, B. (1989a). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Sorbom, D. (1974). A general method for studying differences in factor means and factor structure between groups. British Journal of Mathematical and Statistical Psychology, 27, 229-239.

222