Mplus Short Courses
Day 5A

Multilevel Modeling With Latent Variables Using Mplus

Linda K. Muthén
Bengt Muthén

www.statmodel.com

1

Table Of Contents

General Latent Variable Modeling Framework	4
Complex Survey Data Analysis	11
Intraclass Correlation	12
Design Effects	14
Two-Level Regression Analysis	23
Two-Level Logistic Regression	44
Two-Level Path Analysis	50
Two-Level Factor Analysis	65
SIMS Variance Decomposition	77
Aggression Items	82
Two-Level Factor Analysis With Covariates	86
Multiple Group, Two-Level Factor Analysis	106
Two-Level SEM	122
Practical Issues Related To The Analysis Of Multilevel Data	133
Technical Aspects Of Multilevel Modeling	136
Multivariate Approach To Multilevel Modeling	145
Twin Modeling	150
Multilevel Growth Models	152
Three-Level Modeling	156
Multilevel Discrete-Time Survival Analysis	175
References	180

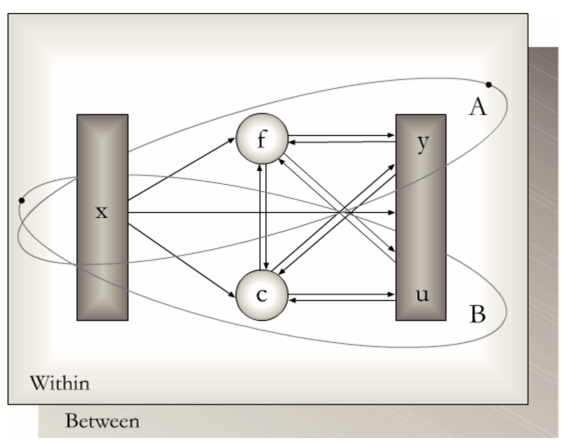
2

Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn

3

General Latent Variable Modeling Framework



4

Mplus

Several programs in one

- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis
- Survival analysis
- Multilevel analysis
- Complex survey data analysis
- Monte Carlo simulation

Fully integrated in the general latent variable framework

Overview

Single-Level Analysis

	Cross-Sectional	Longitudinal
Continuous Observed And Latent Variables	***Day 1*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling	***Day 2*** Growth Analysis
Adding Categorical Observed And Latent Variables	***Day 3*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling Latent Class Analysis Factor Mixture Analysis Structural Equation Mixture Modeling	***Day 4*** Latent Transition Analysis Latent Class Growth Analysis Growth Analysis Growth Mixture Modeling Discrete-Time Survival Mixture Analysis Missing Data Analysis

Overview (Continued)

Multilevel Analysis

	Cross-Sectional	Longitudinal
Continuous Observed And Latent Variables	***Day 5*** Regression Analysis Path Analysis Exploratory Factor Analysis Confirmatory Factor Analysis Structural Equation Modeling	***Day 5*** Growth Analysis
Adding Categorical Observed And Latent Variables	***Day 5*** Latent Class Analysis Factor Mixture Analysis	***Day 5*** Growth Mixture Modeling

7

Analysis With Multilevel Data

Used when data have been obtained by cluster sampling and/or unequal probability sampling to avoid biases in parameter estimates, standard errors, and tests of model fit and to learn about both within- and between-cluster relationships.

Analysis Considerations

- Sampling perspective
  - Aggregated modeling – SUDAAN
    - TYPE = COMPLEX
      - Clustering, sampling weights, stratification (Asparouhov, 2005)

8

Analysis With Multilevel Data (Continued)

- Multilevel perspective
  - Disaggregated modeling – multilevel modeling
    - TYPE = TWOLEVEL
      - Clustering, sampling weights, stratification
  - Multivariate modeling
    - TYPE = GENERAL
      - Clustering, sampling weights, stratification
- Combined sampling and multilevel perspective
  - TYPE = COMPLEX TWOLEVEL
    - Clustering, sampling weights, stratification

Analysis With Multilevel Data (Continued)

Analysis Areas

- Multilevel regression analysis
- Multilevel path analysis
- Multilevel factor analysis
- Multilevel SEM
- Multilevel growth modeling
- Multilevel latent class analysis
- Multilevel latent transition analysis
- Multilevel growth mixture modeling

Complex Survey Data Analysis

11

Intraclass Correlation

Consider nested, random-effects ANOVA for unit i in cluster j,

$$y_{ij} = v + \eta_j + \varepsilon_{ij} \; ; \; i = 1, 2, \ldots, n_j \; ; \; j = 1, 2, \ldots, J. \qquad (44)$$

Random sample of J clusters (e.g. schools).

With timepoint as i and individual as j, this is a repeated measures model with random intercepts.

Consider the covariance and variances for cluster members $i = k$ and $i = l$,

$$Cov(y_{kj}, y_{lj}) = V(\eta), \qquad (45)$$

$$V(y_{kj}) = V(y_{lj}) = V(\eta) + V(\varepsilon), \qquad (46)$$

resulting in the intraclass correlation

$$\rho(y_{kj}, y_{lj}) = V(\eta)/[V(\eta) + V(\varepsilon)]. \qquad (47)$$

Interpretation: Between-cluster variability relative to total variation, intra-cluster homogeneity.

12

NLSY Household Clusters

Household Type (# of respondents)	# of Households*	Intraclass Correlations for Siblings: Year	Heavy Drinking
Single	5,944	1982	0.19
Two	1,985	1983	0.18
Three	634	1984	0.12
Four	170	1985	0.09
Five	32	1988	0.04
Six	5	1989	0.06

Total number of households: 8,770

Total number of respondents: 12,686

Average number of respondents per household: 1.4

*Source: NLS User's Guide, 1994, p.247

13

Design Effects

Consider cluster sampling with equal cluster sizes and the sampling variance of the mean.

V_C : correct variance under cluster sampling
V_{SRS} : variance assuming simple random sampling

$V_C \geq V_{SRS}$ but cluster sampling more convenient, less expensive.

$$DEFF = V_C / V_{SRS} = 1 + (s-1)\,\rho, \qquad (47)$$

where s is the common cluster size and ρ is the intraclass correlation (common range: 0.00 – 0.50).

14

Random Effects ANOVA Example

200 clusters of size 10 with intraclass correlation 0.2 analyzed as:

- TYPE = TWOLEVEL
- TYPE = COMPLEX
- Regular analysis, ignoring clustering

  DEFF = 1 + 9 * 0.2 = 2.8

Input For Two-Level Random Effects ANOVA Analysis

```
TITLE:      Random effects ANOVA data
            Two-level analysis with balanced data

DATA:       FILE = anova.dat;

VARIABLE:   NAMES = y cluster;
            USEV = y;
            CLUSTER = cluster;

ANALYSIS:   TYPE = TWOLEVEL;

MODEL:
            %WITHIN%
            y;
            %BETWEEN%
            y;
```

Output Excerpts Two-Level Random Effects ANOVA Analysis

Model Results

```
                    Estimates      S.E.       Est./S.E.
Within Level
  Variances
     Y                  0.779      0.025         31.293
Between Level
  Means
     Y                  0.003      0.038          0.076
  Variances
     Y                  0.212      0.028          7.496
```

17

Input For Complex Random Effects ANOVA Analysis

```
TITLE:      Random effects ANOVA data
            Complex analysis with balanced data

DATA:       FILE = anova.dat;

VARIABLE:   NAMES = y cluster;
            USEV = y;
            CLUSTER = cluster;

ANALYSIS:   TYPE = COMPLEX;
```

18

Output Excerpts Complex Random Effects ANOVA Analysis

Model Results

	Estimates	S.E.	Est./S.E.
Means			
Y	0.003	0.038	0.076
Variances			
Y	0.990	0.036	27.538

19

Input For Random Effects ANOVA Analysis Ignoring Clustering

```
TITLE:      Random effects ANOVA data
            Ignoring clustering

DATA:       FILE = anova.dat;

VARIABLE:   NAMES = y cluster;
            USEV = y;
!           CLUSTER = cluster;

ANALYSIS:   TYPE = MEANSTRUCTURE;
```

20

Output Excerpts Random Effects ANOVA Analysis Ignoring Clustering

Model Results

```
                    Estimates      S.E.      Est./S.E.
 Means
    Y                   0.003      0.022          0.131
 Variances
    Y                   0.990      0.031         31.623
```

Note: The estimated mean has SE = 0.022 instead of the correct 0.038

Further Readings On Complex Survey Data

Asparouhov, T. (2005). Sampling weights in latent variable modeling. Structural Equation Modeling, 12, 411-434.

Chambers, R.L. & Skinner, C.J. (2003). Analysis of survey data. Chichester: John Wiley & Sons.

Kaplan, D. & Ferguson, A.J (1999). On the utilization of sample weights in latent variable models. Structural Equation Modeling, 6, 305-321.

Korn, E.L. & Graubard, B.I (1999). Analysis of health surveys. New York: John Wiley & Sons.

Patterson, B.H., Dayton, C.M. & Graubard, B.I. (2002). Latent class analysis of complex sample survey data: application to dietary data. Journal of the American Statistical Association, 97, 721-741.

Skinner, C.J., Holt, D. & Smith, T.M.F. (1989). Analysis of complex surveys. West Sussex, England: Wiley.

Stapleton, L. (2002). The incorporation of sample weights into multilevel structural equation models. Structural Equation Modeling, 9, 475-502.

Two-Level Regression Analysis

23

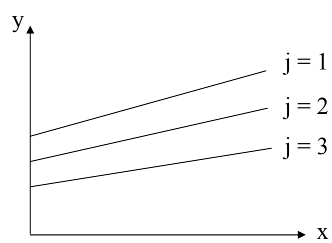
Cluster-Specific Regressions

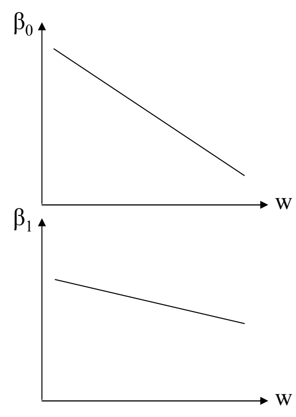
Individual *i* in cluster *j*

(1) $y_{ij} = \beta_{0j} + \beta_{1j}\, x_{ij} + r_{ij}$

(2a) $\beta_{0j} = \gamma_{00} + \gamma_{01}\, w_j + u_{0j}$

(2b) $\beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}$





24

Two-Level Regression Analysis With Random Intercepts And Random Slopes In Multilevel Terms

Two-level analysis (individual i in cluster j):

y_{ij} : individual-level outcome variable
x_{ij} : individual-level covariate
w_j : cluster-level covariate

Random intercepts, random slopes:

$$Level\ 1\ (Within)\ :\ y_{ij} = \beta_{0j} + \beta_{1j}\, x_{ij} + r_{ij}, \tag{1}$$

$$Level\ 2\ (Between)\ :\ \beta_{0j} = \gamma_{00} + \gamma_{01}\, w_j + u_{0j}, \tag{2a}$$

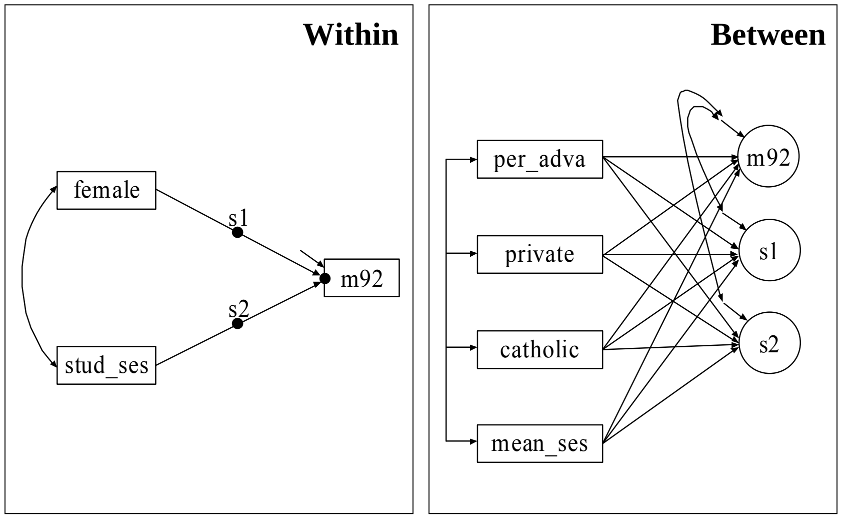
$$Level\ 2\ (Between)\ :\ \beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}. \tag{2b}$$

- Mplus gives the same estimates as HLM/MLwiN ML (not REML):
  - $V(r)$ (residual variance for level 1)
  - $\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}, V(u_0), V(u_1), Cov(u_0, u_1)$
- Centering of x: subtracting grand mean or group (cluster) mean

NELS Data

- The data—National Education Longitudinal Study (NELS:88)
  - Base year Grade 8—followed up in Grades 10 and 12
  - Students sampled within 1,035 schools—approximately 26 students per school, n = 14,217
  - Variables—reading, math, science, history-citizenship-geography, and background variables

NELS Math Achievement Regression



Input For NELS Math Achievement Regression

```
TITLE:     NELS math achievement regression

DATA:      FILE IS completev2.dat;
           ! National Education Longitudinal Study (NELS)
           FORMAT IS f8.0 12f5.2 f6.3 f11.4 23f8.2
           f18.2 f8.0 4f8.2;

VARIABLE:  NAMES ARE school r88 m88 s88 h88 r90 m90 s90 h90 r92
           m92 s92 h92 stud_ses f2pnlwt transfer minor coll_asp
           algebra retain aca_back female per_mino hw_time
           salary dis_fair clas_dis mean_col per_high unsafe
           num_frie teaqual par_invo ac_track urban size rural
           private mean_ses catholic stu_teac per_adva tea_exce
           tea_res;

           USEV = m92 female stud_ses per_adva private catholic
           mean_ses;

           !per_adva = percent teachers with an MA or higher

           WITHIN = female stud_ses;
           BETWEEN = per_adva private catholic mean_ses;
           MISSING = blank;
           CLUSTER = school;
           CENTERING = GRANDMEAN (stud_ses per_adva mean_ses);
```

Input For NELS Math Achievement Regression (Continued)

```
ANALYSIS: TYPE = TWOLEVEL RANDOM MISSING;

MODEL:
          %WITHIN%
          s1 | m92 ON female;
          s2 | m92 ON stud_ses;

          %BETWEEN%
          m92 s1 s2 ON per_adva private catholic mean_ses;
          m92 WITH s1 s2;
OUTPUT:   TECH8 SAMPSTAT;
```


Output Excerpts NELS Math Achievement Regression

N = 10,933

Summary of Data

```
Number of clusters     902

Size (s)  Cluster ID with Size s

 1        89863  75862  52654   1995  32661  89239  56214
 2        41743  81263  45025  26790  60281  82860  56241  21474
           4570  27159  11662  87842  38454
 3        65407  61407  83048  42640  41412  67708  83085  39685
          40402  93469  98582  68595  11517  17543  75498  81069
          66512
 4        31646  68153  85508  26234  83390  60835  74400  20770
           5095  10904  93569  38063  86733  66125  51670  10910
          98461  44395  95317  64112  50880  77381  12835  47555
           9208  93859  35719  67574  20048  34139  25784  80675
 5        14464  74791  18219  10468  72193  97616  15773    877
           9471  83234  68254  68028  70718   3496   6842  45854
```

Output Excerpts NELS Math Achievement Regression (Continued)

22	79570	15426	97947	93599	85125	10926	4603
23	6411	60328	70024	67835			
24	36988	22874	50626	19091			
25	56619	59710	34292	18826	62209		
26	44586	67832	16515				
27	82887						
28	847	76909					
30	36177						
31	12786	53660	47120	94802			
32	80553						
34	53272						
36	89842	31572					
42	99516						
43	75115						

```
Average cluster size  12.187
Estimated Intraclass Correlations for the Y Variables
          Intraclass
Variable Correlation
M92          0.107
```

31

Output Excerpts NELS Math Achievement Regression (Continued)

Tests of Model Fit

```
Loglikelihood
          H0 Value                         -39390.404
Information Criteria
          Number of Free parameters              21
          Akaike (AIC)                     78822.808
          Bayesian (BIC)                   78976.213
          Sample-Size Adjusted BIC         78909.478
             (n* = (n + 2) / 24)
```

Model Results

```
                     Estimates     S.E.   Est./S.E.
 Within Level
 Residual
 Variances
    M92                70.577     1.149    61.442
 Between Level
 S1          ON
    PER_ADVA            0.084     0.841     0.100
    PRIVATE            -0.134     0.844    -0.159
    CATHOLIC           -0.736     0.780    -0.944
    MEAN_SES           -0.232     0.428    -0.542
```

32

Output Excerpts NELS Math Achievement Regression (Continued)

```
S2          ON          Estimates       S.E.    Est./S.E.
   PER_ADVA                1.348       0.521       2.587
   PRIVATE                -1.890       0.706      -2.677
   CATHOLIC               -1.467       0.562      -2.612
   MEAN_SES                1.031       0.283       3.640
M92         ON
   PER_ADVA                0.195       0.727       0.268
   PRIVATE                 1.505       1.108       1.358
   CATHOLIC                0.765       0.650       1.178
   MEAN_SES                3.912       0.399       9.814
S1           WITH
   M92                    -4.456       1.007      -4.427
 S2          WITH
   M92                     0.128       0.399       0.322
Intercepts
   M92                    55.136       0.185     297.248
   S1                     -0.819       0.211      -3.876
   S2                      4.841       0.152      31.900
 Residual Variances
   M92                     8.679       1.003       8.649
   S1                      5.740       1.411       4.066
   S2                      0.307       0.527       0.583
```

33

Cross-Level Influence

Between-level (level 2) variable *w* influencing within-level (level 1) *y* variable:

Random intercept

$$y_{ij} = \beta_{0j} + \beta_1 x_{ij} + r_{ij}$$

$$\beta_{0j} = \overbrace{\gamma_{00} + \gamma_{01} w_j + u_{0j}}$$

Mplus:

```
MODEL:
        %WITHIN%;
        y ON x;    ! estimates beta1
        %BETWEEN%;
        y ON w;    ! y is the same as beta0
                   ! estimates gamma01
```

34

Cross-Level Influence (Continued)

Cross-level interaction, or between-level (level 2) variable moderating a within level (level 1) relationship:

Random slope

$$y_{ij} = \beta_0 + \beta_{1j}\, x_{ij} + r_{ij}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}$$

Mplus:

```
MODEL:

        %WITHIN%;
        beta1 | y ON x;
        %BETWEEN%;
        beta1 ON w;      ! estimates gamma11
```

35

Random Slopes

- In single-level modeling random slopes β_i describe variation across individuals *i*,

$$y_i = \alpha_i + \beta_i x_i + \varepsilon_i, \quad (100)$$

$$\alpha_i = \alpha + \zeta_{0i}, \quad (101)$$

$$\beta_i = \beta + \zeta_{1i}, \quad (102)$$

resulting in heteroscedastic residual variances

$$V(\, y_i \mid x_i) = V(\beta_i)\; x_i^2 + \theta. \quad (103)$$

- In two-level modeling random slopes β_j describe variation across clusters *j*

$$y_{ij} = a_j + \beta_j\, x_{ij} + \varepsilon_{ij}, \quad (104)$$

$$a_j = a + \zeta_{0j}, \quad (105)$$

$$\beta_j = \beta + \zeta_{1j}, \quad (106)$$

A small variance for a random slope typically leads to slow convergence of the ML-EM iterations. This suggests respecifying the slope as fixed.

Mplus allows random slopes for predictors that are
- Observed covariates
- Observed dependent variables
- Continuous latent variables

36

Further Readings On Multilevel Regression Analysis

Ludtke Marsh, Robitzsch, Trautwein, Asparouhov, Muthen (2007). Analysis of group level effects using multilevel modeling: Probing a latent covariate approach. Submitted for publication.

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

37

Logistic And Probit Regression

38

Categorical Outcomes: Logit And Probit Regression

Probability varies as a function of x variables (here x_1, x_2)

$$P(u = 1 \mid x_1, x_2) = F[\beta_0 + \beta_1 x_1 + \beta_2 x_2], \qquad (22)$$

$P(u = 0 \mid x_1, x_2) = 1 - P[u = 1 \mid x_1, x_2]$, where $F[z]$ is either the standard normal ($\Phi[z]$) or logistic ($1/[1 + e^{-z}]$) distribution function.

Example: Lung cancer and smoking among coal miners

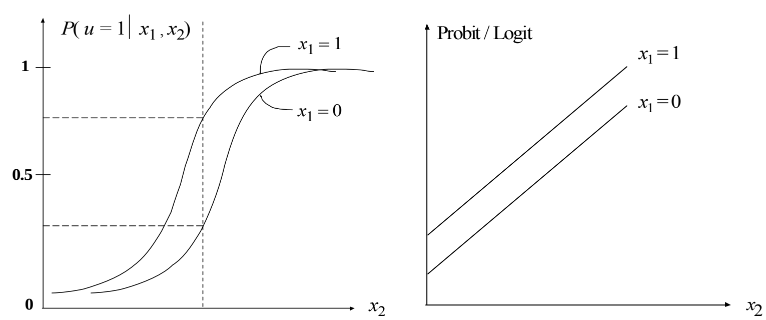
u lung cancer ($u = 1$) or not ($u = 0$)

x_1 smoker ($x_1 = 1$), non-smoker ($x_1 = 0$)

x_2 years spent in coal mine

Categorical Outcomes: Logit And Probit Regression

$$P(u = 1 \mid x_1, x_2) = F[\beta_0 + \beta_1 x_1 + \beta_2 x_2], \qquad (22)$$



Interpreting Logit And Probit Coefficients

- Sign and significance
- Odds and odds ratios
- Probabilities

Logistic Regression And Log Odds

Odds $(u = 1 \mid x) = P(u = 1 \mid x) \,/\, P(u = 0 \mid x)$
$= P(u = 1 \mid x) \,/\, (1 - P(u = 1 \mid x))$.

The logistic function

$$P(u = 1 \mid x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

gives a log odds linear in x,

$$logit = log\,[odds\,(u = 1 \mid x)] = log\,[P(u = 1 \mid x) \,/\, (1 - P(u = 1 \mid x))]$$

$$= \log \left[\frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} \;/\; (1 - \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}) \right]$$

$$= \log \left[\frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} * \frac{1 + e^{-(\beta_0 + \beta_1 x)}}{e^{-(\beta_0 + \beta_1 x)}} \right]$$

$$= \log \left[e^{(\beta_0 + \beta_1 x)} \right] = \beta_0 + \beta_1 x$$

Logistic Regression And Log Odds (Continued)

- $logit = log\ odds = \beta_0 + \beta_1\ x$
- When x changes one unit, the *logit* (*log odds*) changes β_1 units
- When x changes one unit, the *odds* changes e^{β_1} units

43

Two-Level Logistic Regression

With j denoting cluster,

$\text{logit}_{ij} = \log\ (P(u_{ij} = 1)/P(u_{ij} = 0)) = \alpha_j + \beta_j * x_{ij}$

where

$\alpha_j = \alpha + u_{0j}$
$\beta_j = \beta + u_{1j}$

High/low α_j value means high/low logit (high log odds)

44

Predicting Juvenile Delinquency From First Grade Aggressive Behavior

- Cohort 1 data from the Johns Hopkins University Preventive Intervention Research Center
- n= 1,084 students in 40 classrooms, Fall first grade
- Covariates: gender and teacher-rated aggressive behavior

45

Input For Two-Level Logistic Regression

```
TITLE:
        Hopkins Cohort 1 2-level logistic regression
DATA:
        FILE = Cohort1_classroom_ALL.DAT;
VARIABLE:

        NAMES =  prcid juv99 gender stub1F bkRule1F harmO1F
                 bkThin1F yell1F takeP1F fight1F lies1F
                 tease1F;
        CLUSTER = classrm;
        USEVAR = juv99 male aggress;
        CATEGORICAL =  juv99;
        MISSING = ALL (999);
        WITHIN = male aggress;
DEFINE:
        male = 2 - gender;
        aggress = stub1F + bkRule1F + harmO1F + bkThin1F +
                 yell1F + takeP1F + fight1F + lies1F + tease1F;
```

46

Input For Two-Level Logistic Regression (Continued)

```
ANALYSIS:
        TYPE = TWOLEVEL MISSING;
        PROCESS = 2;
MODEL:
        %WITHIN%
        juv99 ON male aggress;
        %BETWEEN%
OUTPUT:
        TECH1 TECH8;
```


Output Excerpts Two-Level Logistic Regression

```
MODEL RESULTS

                    Estimates    S.E    Est./S.E.
Within Level
  JUV99         ON
    MALE              1.071     0.149     7.193
    AGGRESS           0.060     0.010     6.191

Between Level
Thresholds
  JUV99$1             2.981     0.205    14.562

Variances
  JUV99               0.807     0.250     3.228
```

Understanding The Between-Level Intercept Variance

- Intra-class correlation
  - ICC = $0.807/(\pi^2/3 + 0.807)$
- Odds ratios
  - Larsen & Merlo (2005). Appropriate assessment of neighborhood effects on individual health: Integrating random and fixed effects in multilevel logistic regression. American Journal of Epidemiology, 161, 81-88.
  - Larsen proposes MOR:
    "Consider two persons with the same covariates, chosen randomly from two different clusters. The MOR is the median odds ratio between the person of higher propensity and the person of lower propensity."

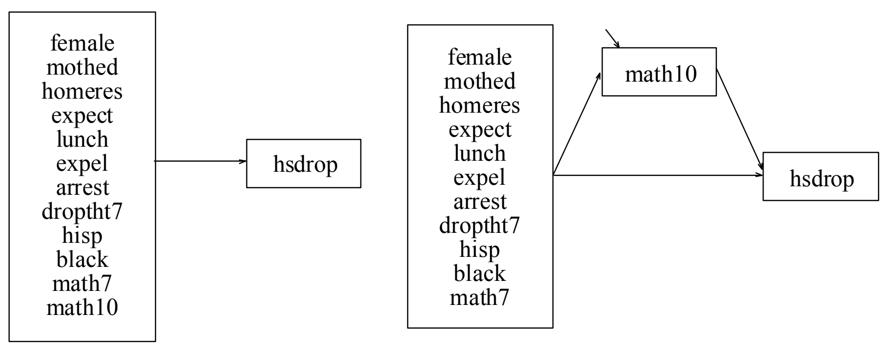
    MOR = $\exp(\sqrt{(2 * \sigma^2)} * \Phi^{-1} (0.75))$

    In the current example, ICC = 0.20, MOR = 2.36
- Probabilities
  - Compare α_j=1 SD and α_k=-1 SD from the mean

Two-Level Path Analysis

A Path Model With A Binary Outcome And A Mediator With Missing Data

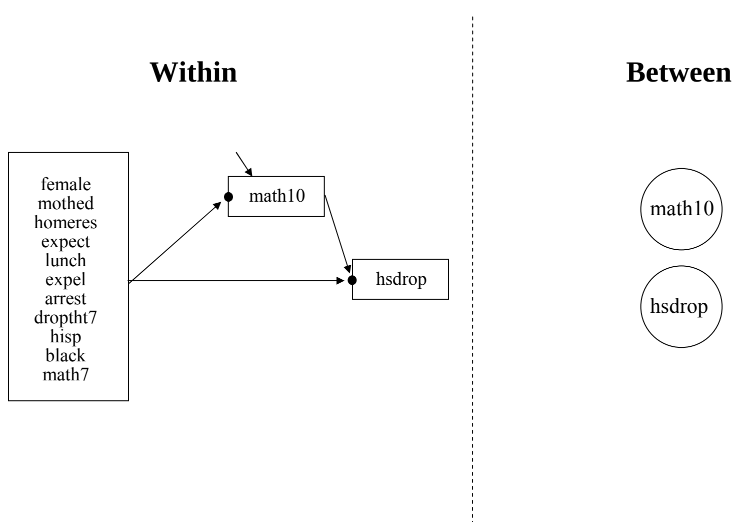
Logistic Regression | **Path Model**



51

Two-Level Path Analysis

Within | **Between**



52

Input For A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable

```
TITLE:      a twolevel path analysis with a categorical outcome
            and missing data on the mediating variable
DATA:       FILE = lsayfull_dropout.dat;
VARIABLE:   NAMES = female mothed homeres math7 math10 expel
            arrest hisp black hsdrop expect lunch droptht7
            schcode;
            MISSING = ALL (9999);
            CATEGORICAL = hsdrop;
            CLUSTER = schcode;
            WITHIN = female mothed homeres expect math7 lunch
            expel arrest droptht7 hisp black;
ANALYSIS:   TYPE = TWOLEVEL MISSING;
            ESTIMATOR = ML;
            ALGORITHM = INTEGRATION;
            INTEGRATION = MONTECARLO (500);
```

53

Input For A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

```
MODEL:
            %WITHIN%
            hsdrop ON female mothed homeres expect math7 math10
            lunch expel arrest droptht7 hisp black;
            math10 ON female mothed homeres expect math7 lunch
            expel arrest droptht7 hisp black;

            %BETWEEN%
            hsdrop*1; math10*1;

OUTPUT:     PATTERNS SAMPSTAT STANDARDIZED TECH1 TECH8;
```

54

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable

Summary Of Data

```
          Number of patterns          2
          Number of clusters         44

     Size (s)        Cluster ID with Size s
        12             304
        13             305
        36             307     122
        38             106     112
        39             138     109
        40             103
        41             308
        42             146     120
        43             102     101
        44             303     143
        45             141
```

55

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

```
     Size (s)        Cluster ID with Size s
        46             144
        47             140
        49             108
        50             126     111     110
        51             127     124
        52             137     117     147     118     301     136
        53             142     131
        55             145     123
        57             135     105
        58             121
        59             119
        73             104
        89             302
        93             309
       118             115
```

56

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Within Level					
HSDROP ON					
FEMALE	0.323	0.171	1.887	0.323	0.077
MOTHED	-0.253	0.103	-2.457	-0.253	-0.121
HOMERES	-0.077	0.055	-1.401	-0.077	-0.061
EXPECT	-0.244	0.065	-3.756	-0.244	-0.159
MATH7	-0.011	0.015	-0.754	-0.011	-0.055
MATH10	-0.031	0.011	-2.706	-0.031	-0.197
LUNCH	0.008	0.006	1.324	0.008	0.074
EXPEL	0.947	0.225	4.201	0.947	0.121
ARREST	0.068	0.321	0.212	0.068	0.007
DROPTHT7	0.757	0.284	2.665	0.757	0.074
HISP	-0.118	0.274	-0.431	-0.118	-0.016
BLACK	-0.086	0.253	-0.340	-0.086	-0.013

57

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
MATH10 ON					
FEMALE	-0.841	0.398	-2.110	-0.841	-0.031
MOTHED	0.263	0.215	1.222	0.263	0.020
HOMERES	0.568	0.136	4.169	0.568	0.070
EXPECT	0.985	0.162	6.091	0.985	0.100
MATH7	0.940	0.023	40.123	0.940	0.697
LUNCH	-0.039	0.017	-2.308	-0.039	-0.059
EXPEL	-1.293	0.825	-1.567	-1.293	-0.026
ARREST	-3.426	1.022	-3.353	-3.426	-0.054
DROPTHT7	-1.424	1.049	-1.358	-1.424	-0.022
HISP	-0.501	0.728	-0.689	-0.501	-0.010
BLACK	-0.369	0.733	-0.503	-0.369	-0.009

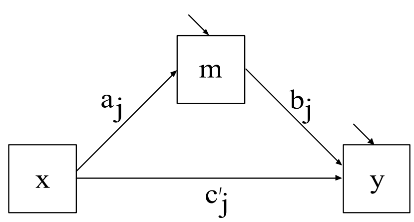
58

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

```
                         Estimates       S.E.  Est./S.E.    Std      StdYX

 Residual Variances
    MATH10                 62.010      2.162     28.683   62.010     0.341

 Between Level
  Means
    MATH10                 10.226      1.340      7.632   10.226     5.276
 Thresholds
    HSDROP$1               -1.076      0.560     -1.920
 Variances
    HSDROP                  0.286      0.133      2.150    0.286     1.000
    MATH10                  3.757      1.248      3.011    3.757     1.000
```

59

Two-Level Mediation



Indirect effect:

$$\alpha + \beta + Cov\,(a_j, b_j)$$

Bauer, Preacher & Gil (2006). Conceptualizing and testing random indirect effects and moderated mediation in multilevel models: New procedures and recommendations. Psychological Methods, 11, 142-163.

60

Input For Two-Level Mediation

```
MONTECARLO:
        NAMES ARE y m x;
        WITHIN = x;
        NOBSERVATIONS = 1000;
        NCSIZES = 1;
        CSIZES = 100 (10);
        NREP = 100;

MODEL POPULATION:
        %WITHIN%
        c | y ON x;
        b | y ON m;
        a | m ON x;
        x*1; m*1; y*1;
        %BETWEEN%
        y WITH m*0.1 b*0.1 a*0.1 c*0.1;
        m WITH b*0.1 a*0.1 c*0.1;
        a WITH b*0.1 c*0.1;
        b WITH c*0.1;
        y*1 m*1 a*1 b*1 c*1;
        [a*0.4 b*0.5 c*0.6];
```

61

Input For Two-Level Mediation (Continued)

```
ANALYSIS:
        TYPE = TWOLEVEL RANDOM;
MODEL:
        %WITHIN%
        c | y ON x;
        b | y ON m;
        a | m ON x;
        m*1; y*1;
        %BETWEEN%
        y WITH M*0.1 b*0.1 a*0.1 c*0.1;
        m WITH b*0.1 a*0.1 c*0.1;
        a WITH b*0.1 (cab);
        a WITH c*0.1;
        b WITH c*0.1;
        y*1 m*1 a*1 b*1 c*1;
        [a*0.4] (ma);
        [b*0.5] (mb);
        [c*0.6];

 MODEL CONSTRAINT:
        NEW(m*0.3);
        m=ma*mb+cab;
```

62

Output Excerpts Two Level Mediation

	Estimates Population	Average	Std.Dev.	S.E. Average	M. S. E.	95% Cover	% Sig Coeff
Within Level							
Residual variances							
Y	1.000	1.0020	0.0530	0.0530	0.0028	0.960	1.000
M	1.000	1.0011	0.0538	0.0496	0.0029	0.910	1.000
Between Level							
Y WITH							
B	0.100	0.1212	0.1246	0.114	0.0158	0.910	0.210
A	0.100	0.1086	0.1318	0.1162	0.0173	0.910	0.190
C	0.100	0.0868	0.1121	0.1237	0.0126	0.940	0.090
M WITH							
B	0.100	0.1033	0.1029	0.1085	0.0105	0.940	0.120
A	0.100	0.0815	0.1081	0.1116	0.0119	0.950	0.070
C	0.100	0.1138	0.1147	0.1165	0.0132	0.970	0.160
A WITH							
B	0.100	0.0964	0.1174	0.1101	0.0137	0.920	0.150
C	0.100	0.0756	0.1376	0.1312	0.0193	0.910	0.110

Output Excerpts Two-Level Mediation (Continued)

B WITH							
C	0.100	0.0892	0.1056	0.1156	0.0112	0.960	0.070
Y WITH							
M	0.100	0.1034	0.1342	0.1285	0.0178	0.940	0.140
Means							
Y	0.000	0.0070	0.1151	0.1113	0.0132	0.950	0.050
M	0.000	-0.0031	0.1102	0.1056	0.0120	0.950	0.050
C	0.600	0.5979	0.1229	0.1125	0.0150	0.930	1.000
B	0.500	0.5022	0.1279	0.1061	0.0162	0.890	1.000
A	0.400	0.3854	0.0972	0.1072	0.0096	0.970	0.970
Variances							
Y	1.000	1.0071	0.1681	0.1689	0.0280	0.910	1.000
M	1.000	1.0113	0.1782	0.1571	0.0316	0.930	1.000
C	1.000	0.9802	0.1413	0.1718	0.0201	0.980	1.000
B	1.000	0.9768	0.1443	0.1545	0.0212	0.950	1.000
A	1.000	1.0188	0.1541	0.1587	0.0239	0.950	1.000
New/Additional Parameters							
M	0.300	0.2904	0.1422	0.1316	0.0201	0.950	0.550

Two-Level Factor Analysis

65

Two-Level Factor Analysis

- Recall random effects ANOVA (individual i in cluster j):

$$y_{ij} = v + \eta_j + \varepsilon_{ij} = y_{B_j} + y_{W_{ij}}$$

- Two-level factor analysis (r = 1, 2, …, p items):

$$y_{rij} = v_r + \lambda_{B_r}\, \eta_{B_j} + \varepsilon_{B_{rj}} + \lambda_{W_r}\, \eta_{Wij} + \varepsilon_{Wrij}$$

(between-cluster variation) (within-cluster variation)

66

Two-Level Factor Analysis (Continued)

- Covariance structure:

$$V(\boldsymbol{y}) = V(\boldsymbol{y}_B) + V(\boldsymbol{y}_w) = \boldsymbol{\Sigma}_B + \boldsymbol{\Sigma}_w,$$

$$\boldsymbol{\Sigma}_B = \boldsymbol{\Lambda}_B \ \boldsymbol{\Psi}_B \ \boldsymbol{\Lambda}_B{}' + \boldsymbol{\Theta}_B,$$

$$\boldsymbol{\Sigma}_W = \boldsymbol{\Lambda}_W \ \boldsymbol{\Psi}_W \ \boldsymbol{\Lambda}_W{}' + \boldsymbol{\Theta}_W \,.$$

- Two interpretations:
  - variance decomposition, including decomposing the residual
  - random intercept model

67

Two-Level Factor Analysis And Design Effects

Muthén & Satorra (1995; Sociological Methodology): Monte Carlo study using two-level data (200 clusters of varying size and varying intraclass correlations), a latent variable model with 10 variables, 2 factors, conventional ML using the regular sample covariance matrix S_T, and 1,000 replications (d.f. = 34).

$$\Lambda_B = \Lambda_W = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad \Psi_B, \Theta_B \text{ reflecting different icc's}$$

$$y_{ij} = \nu + \Lambda(\eta_{B_j} + \eta_{W_{ij}}) + \varepsilon_{B_j} + \varepsilon_{W_{ij}}$$

$$V(y) = \Sigma_B + \Sigma_W = \Lambda(\Psi_B + \Psi_W)\,\Lambda' + \Theta_B + \Theta_W$$

68

Two-Level Factor Analysis And Design Effects (Continued)

Inflation of χ^2 due to clustering

Intraclass Correlation		Cluster Size 7	15	30	60
0.05					
	Chi-square mean	35	36	38	41
	Chi-square var	68	72	80	96
	5%	5.6	7.6	10.6	20.4
	1%	1.4	1.6	2.8	7.7
0.10					
	Chi-square mean	36	40	46	58
	Chi-square var	75	89	117	189
	5%	8.5	16.0	37.6	73.6
	1%	1.0	5.2	17.6	52.1
0.20					
	Chi-square mean	42	52	73	114
	Chi-square var	100	152	302	734
	5%	23.5	57.7	93.1	99.9
	1%	8.6	35.0	83.1	99.4

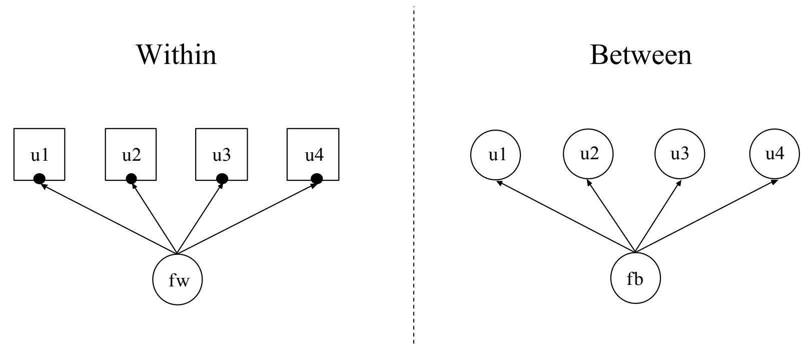
69

Two-Level Factor Analysis And Design Effects (Continued)

- Regular analysis, ignoring clustering
  - Inflated chi-square, underestimated SE's
- TYPE = COMPLEX
  - Correct chi-square and SE's but only if model aggregates, e.g. $\Lambda_B = \Lambda_W$
- TYPE = TWOLEVEL
  - Correct chi-square and SE's

70

Two-Level Factor Analysis (IRT)



$$u^*_{ij} = \lambda\,(f_{B_j} + f_{w_{ij}}) + \varepsilon_{ij}$$

71

Input For A Two-Level Factor Analysis (IRT) Model With Categorical Outcomes

```
TITLE:      this is an example of a two-level factor analysis
            model with categorical outcomes
DATA:       FILE = catrep1.dat;
VARIABLE:   NAMES ARE u1-u6 clus;
            CATEGORICAL = u1-u6;
            CLUSTER = clus;
ANALYSIS:   TYPE = TWOLEVEL;
            ESTIMATION = ML;
            ALGORITHM = INTEGRATION;
MODEL:
            %WITHIN%
            fw BY u1@1
            u2 (1)
            u3 (2)
            u4 (3)
            u5 (4)
            u6 (5);
```

72

Input For A Two-Level Factor Analysis (IRT) Model With Categorical Outcomes (Continued)

```
            %BETWEEN%
            fb BY u1@1
            u2 (1)
            u3 (2)
            u4 (3)
            u5 (4)
            u6 (5);
OUTPUT:     TECH1 TECH8;
```


Output Excerpts A Two-Level Factor Analysis (IRT) Model With Categorical Outcomes

Tests Of Model Fit

```
Loglikelihood

             H0 Value                        -3696.117
Information Criteria

             Number of Free Parameters              13
             Akaike (AIC)                     7418.235
             Bayesian (BIC)                   7481.505
             Sample-Size Adjusted BIC         7440.217
                (n* = (n + 2) / 24)
```

Output Excerpts A Two-Level Factor Analysis (IRT) Model With Categorical Outcomes (Continued)

Model Results

```
                    Estimates       S.E.  Est./S.E.
Within Level
  FW       BY
     U1                 1.000      0.000      0.000
     U2                 0.915      0.146      6.264
     U3                 1.087      0.169      6.437
     U4                 1.058      0.164      6.441
     U5                 1.191      0.185      6.449
     U6                 1.143      0.178      6.439

  Variances
     FW                 0.834      0.191      4.360
```

75

Output Excerpts Two-Level Factor Analysis (IRT) Model With Categorical Outcomes (Continued)

```
                    Estimates       S.E.  Est./S.E.
Between Level
  FB       BY
     U1                 1.000      0.000      0.000
     U2                 0.915      0.146      6.264
     U3                 1.087      0.169      6.437
     U4                 1.058      0.164      6.441
     U5                 1.191      0.185      6.449
     U6                 1.143      0.178      6.439

  Thresholds
     U1$1              -0.206      0.096     -2.150
     U2$1               0.001      0.091      0.007
     U3$1              -0.016      0.100     -0.156
     U4$1              -0.064      0.098     -0.652
     U5$1              -0.033      0.105     -0.315
     U6$1              -0.021      0.102     -0.209

  Variances
     FB                 0.496      0.139      3.562
```

76

SIMS Variance Decomposition

The Second International Mathematics Study (SIMS; Muthén, 1991, JEM).

- National probability sample of school districts selected proportional to size; a probability sample of schools selected proportional to size within school district, and two classes randomly drawn within each school
- 3,724 students observed in 197 classes from 113 schools with class sizes varying from 2 to 38; typical class size of around 20
- Eight variables corresponding to various areas of eighth-grade mathematics
- Same set of items administered as a pretest in the Fall of eighth grade and as a posttest in the Spring.

77

SIMS Variance Decomposition (Continued)

Muthén (1991). Multilevel factor analysis of class and student achievement components. Journal of Educational Measurement, 28, 338-354.

- Research questions: "The substantive questions of interest in this article are the variance decomposition of the subscores with respect to within-class student variation and between-class variation and the change of this decomposition from pretest to posttest. In the SIMS … such variance decomposition relates to the effects of tracking and differential curricula in eighth-grade math. On the one hand, one may hypothesize that effects of selection and instruction tend to increase between-class variation relative to within-class variation, assuming that the classes are homogeneous, have different performance levels to begin with, and show faster growth for higher initial performance level. On the other hand, one may hypothesize that eighth-grade exposure to new topics will increase individual differences among students within each class so that posttest within-class variation will be sizable relative to posttest between-class variation."

78

SIMS Variance Decomposition (Continued)

$y_{rij} = \nu_r + \lambda_{Br}\,\eta_{Bj} + \varepsilon_{Brj} + \lambda_{wr}\,\eta_{wij} + \varepsilon_{wrij}$

$V(y_{rij}) =$ BF + BE + WF + WE

Between reliability: BF / (BF + BE)
– BE often small (can be fixed at 0)

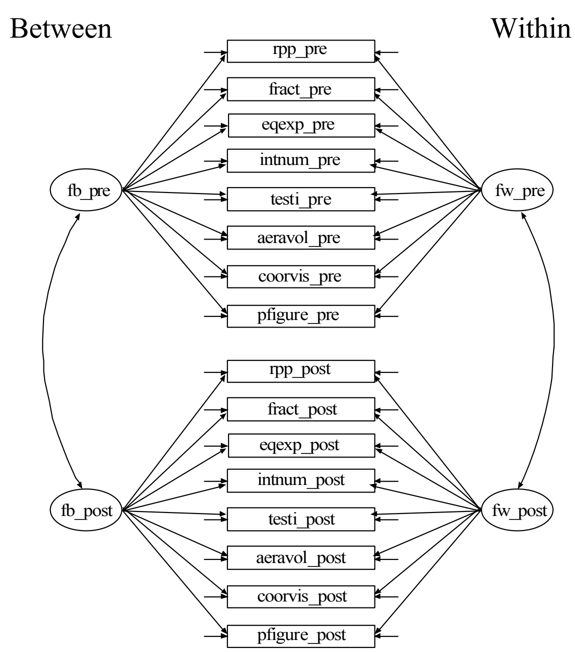
Within reliability: WF / (WF + WE)
– sum of a small number of items gives a large WE

Intraclass correlation:
ICC = (BF + BE) / (BF + BE + WF+ WE)

Large measurement error → large WE → small ICC

True ICC = BF / (BF + WF)

79



80

Table 4: Variance Decomposition of SIMS Achievement Scores (percentages of total variance in parenthesis)

		ANOVA								FACTOR ANALYSIS			
		Pretest			Posttest			% Increase In Variance		Error-free Prop. Between		Error-free % Increase In Variance	
	Number of Items	Between	Within	Prop-Between	Between	Within	Prop-Between	Between	Within	Pre	Post	Between	Within
RPP	8	1.542 (34.0)	2.990 (66.0)	.34	2.084 (38.5)	3.326 (61.5)	.38	35	11	.54	.52	29	41
FRACT	8	1.460 (38.2)	2.366 (61.8)	.38	1.906 (40.8)	2.767 (59.2)	.41	31	17	.60	.58	29	41
EQEXP	6	.543 (26.9)	1.473 (73.1)	.27	1.041 (38.7)	1.646 (61.3)	.39	92	18	.65	.64	113	117
INTNUM	2	.127 (25.2)	.358 (70.9)	.29	.195 (30.6)	.442 (69.4)	.31	54	24	.63	.61	29	41
TESTI	5	.580 (33.3)	1.163 (66.7)	.33	.664 (34.5)	1.258 (65.5)	.34	15	8	.58	.56	29	41
AREAVOL	2	.094 (17.2)	.451 (82.8)	.17	.156 (24.1)	.490 (75.9)	.24	66	9	.54	.52	29	41
COORVIS	3	.173 (20.9)	.656 (79.1)	.21	.275 (28.7)	.680 (68.3)	.32	59	4	.57	.55	29	41
PFIGURE	5	.363 (22.9)	1.224 (77.1)	.23	.711 (42.9)	1.451 (67.1)	.33	96	19	.60	.54	87	136

Second-Generation JHU PIRC Trial Aggression Items

Item Distributions for Cohort 3: Fall 1st Grade (n=362 males in 27 classrooms)

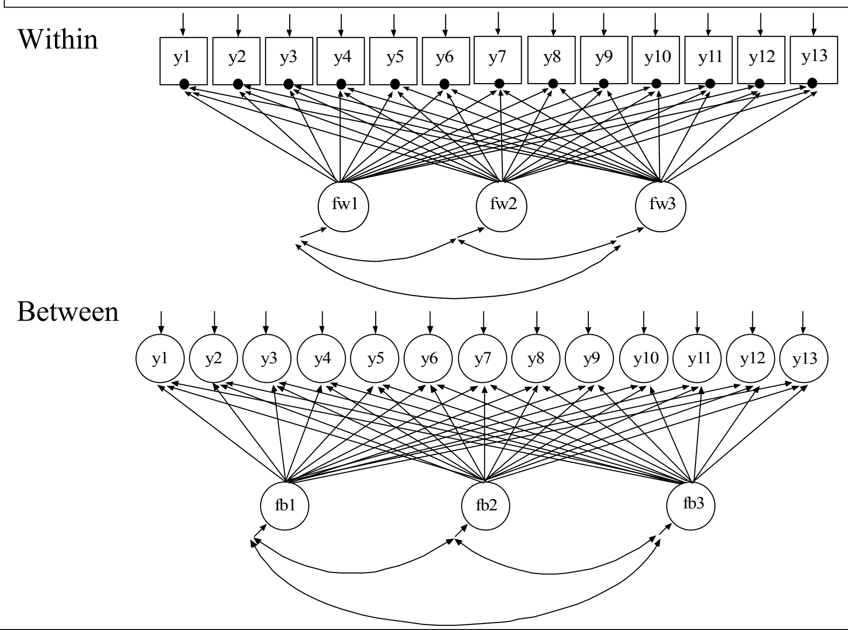
	Almost Never (*scored as 1*)	*Rarely* (*scored as 2*)	*Sometimes* (*scored as 3*)	*Often* (*scored as 4*)	*Very Often* (*scored as 5*)	*Almost Always* (*scored as 6*)
Stubborn	42.5	21.3	18.5	7.2	6.4	4.1
Breaks Rules	37.6	16.0	22.7	7.5	8.3	8.0
Harms Others	69.3	12.4	9.40	3.9	2.5	2.5
Breaks Things	79.8	6.60	5.20	3.9	3.6	0.8
Yells at Others	61.9	14.1	11.9	5.8	4.1	2.2
Takes Others' Property	72.9	9.70	10.8	2.5	2.2	1.9
Fights	60.5	13.8	13.5	5.5	3.0	3.6
Harms Property	74.9	9.90	9.10	2.8	2.8	0.6
Lies	72.4	12.4	8.00	2.8	3.3	1.1
Talks Back to Adults	79.6	9.70	7.80	1.4	0.8	1.4
Teases Classmates	55.0	14.4	17.7	7.2	4.4	1.4
Fights With Classmates	67.4	12.4	10.2	5.0	3.3	1.7
Loses Temper	61.6	15.5	13.8	4.7	3.0	1.4

Hypothesized Aggressiveness Factors

- Verbal aggression
  - Yells at others
  - Talks back to adults
  - Loses temper
  - Stubborn
- Property aggression
  - Breaks things
  - Harms property
  - Takes others' property
  - Harms others
- Person aggression
  - Fights
  - Fights with classmates
  - Teases classmates

83

Two-Level Factor Analysis



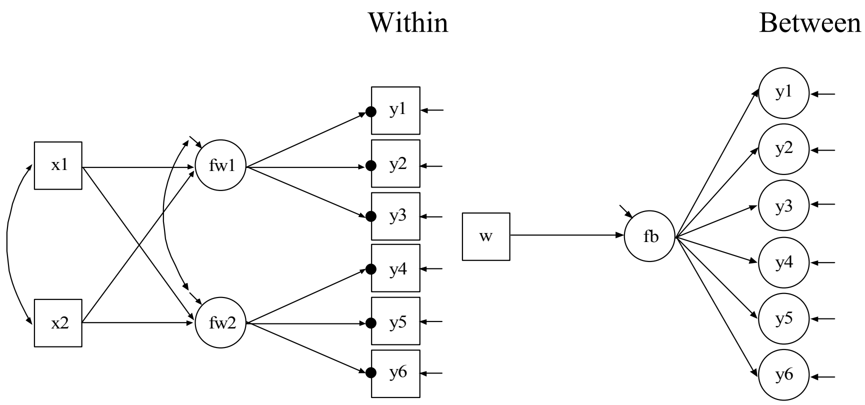
84

Promax Rotated Loadings

	Within-Level Loadings			Between-Level Loadings		
	1	**2**	**3**	**1**	**2**	**3**
Stubborn	0.07	**0.70**	0.05	-0.19	**1.03**	0.07
Breaks Rules	0.25	0.31	0.37	0.15	0.28	0.31
Harms Others	**0.52**	0.16	0.27	0.35	-0.20	**0.72**
Breaks Things	**0.84**	0.16	-0.01	**0.71**	0.01	0.41
Yells at Others	0.15	**0.64**	0.13	0.38	**0.74**	-0.01
Takes Others' Property	**0.57**	0.00	0.37	**0.86**	-0.04	0.12
Fights	0.20	0.21	**0.63**	0.09	0.03	**0.89**
Harms Property	**0.73**	0.21	0.10	**0.90**	-0.05	0.16
Lies	0.48	0.28	0.24	**0.86**	0.33	-0.21
Talks Back to Adults	0.29	**0.71**	0.23	0.41	0.58	-0.04
Teases Classmates	0.11	0.19	**0.62**	0.37	0.31	0.30
Fights With Classmates	0.10	0.31	**0.63**	-0.19	0.38	**0.88**
Loses Temper	0.12	**0.75**	0.04	0.17	**0.78**	0.12

Two-Level Factor Analysis With Covariates

Two-Level Factor Analysis With Covariates



87

Input For Two-Level Factor Analysis With Covariates

```
TITLE:     this is an example of a two-level CFA with
           continuous factor indicators with two factors on the
           within level and one factor on the between level
DATA:      FILE IS ex9.8.dat;
VARIABLE:  NAMES ARE y1-y6 x1 x2 w clus;
           WITHIN = x1 x2;
           BETWEEN = w;
           CLUSTER IS clus;
ANALYSIS:  TYPE IS TWOLEVEL;
MODEL:     %WITHIN%
           fw1 BY y1-y3;
           fw2 BY y4-y6;
           fw1 ON x1 x2;
           fw2 ON x1 x2;
           %BETWEEN%
           fb BY y1-y6;
           fb ON w;
```

88

Input For Monte Carlo Simulations For Two-Level Factor Analysis With Covariates

```
TITLE:        This is an example of a two-level CFA with
              continuous factor indicators with two
              factors on the within level and one factor
              on the between level
MONTECARLO:
              NAMES ARE y1-y6 x1 x2 w;
              NOBSERVATIONS = 1000;
              NCSIZES = 3;
              CSIZES = 40 (5) 50 (10) 20 (15);
              SEED = 58459;
              NREPS = 1;
              SAVE = ex9.8.dat;
              WITHIN = x1 x2;
              BETWEEN = w;
ANALYSIS:     TYPE = TWOLEVEL;
```

89

Input For Monte Carlo Simulations For Two-Level Factor Analysis With Covariates (Continued)

```
MODEL POPULATION:
              %WITHIN%
              x1-x2@1;
              fw1 BY y1@1 y2-y3*1;
              fw2 BY y4@1 y5-y6*1;
              fw1-fw2*1;
              y1-y6*1;
              fw1 ON x1*.5 x2*.7;
              fw2 ON x1*.7 x2*.5;

              %BETWEEN%
              [w@0]; w*1;
              fb BY y1@1 y2-y6*1;
              y1-y6*.3;
              fb*.5;
              fb ON w*1;
```

90

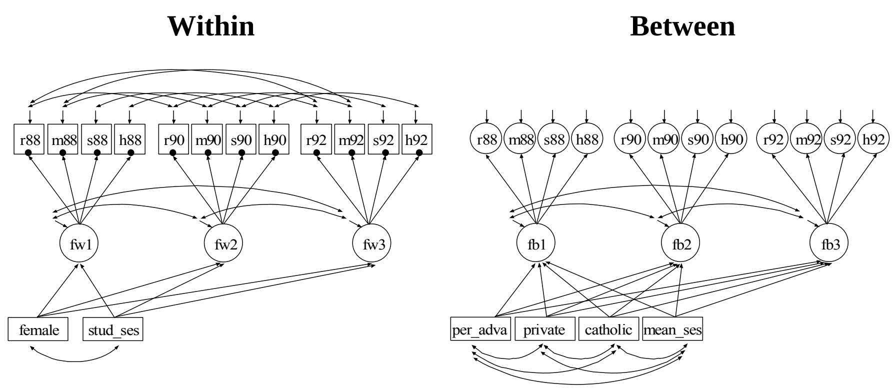
Input For Monte Carlo Simulations For Two-Level Factor Analysis With Covariates (Continued)

```
MODEL:
              %WITHIN%

              fw1 BY y1@1 y2-y3*1;
              fw2 BY y4@1 y5-y6*1;
              fw1-fw2*1;
              y1-y6*1;
              fw1 ON x1*.5 x2*.7;
              fw2 ON x1*.7 x2*.5;

              %BETWEEN%

              fb BY y1@1 y2-y6*1;
              y1-y6*.3;
              fb*.5;
              fb ON w*1;
OUTPUT:
              TECH8 TECH9;
```


NELS Data

- The data—National Education Longitudinal Study (NELS:88)
  - Base year Grade 8—followed up in Grades 10 and 12
  - Students sampled within 1,035 schools—approximately 26 students per school, n = 14,217
  - Variables—reading, math, science, history-citizenship-geography, and background variables
- Data for the analysis—reading, math, science, history-citizenship-geography

NELS Two-Level Longitudinal Factor Analysis With Covariates



Input For NELS Two-Level Longitudinal Factor Analysis With Covariates

```
TITLE:        two-level factor analysis with covariates using the NELS
              data
DATA:         FILE = NELS.dat;
              FORMAT = 2f7.0 f11.4 12f5.2 11f8.2;
VARIABLE:     NAMES = id school f2pnlwt r88 m88 s88 h88 r90 m90 s90 h90
              r92 m92 s92 h92 stud_ses female per_mino urban size rural
              private mean_ses catholic stu_teac per_adva;
                  !Variable Description
                  !m88 = math IRT score in 1988
                  !m90 = math IRT score in 1990
                  !m92 = math IRT score in 1992
                  !r88 = reading IRT score in 1988
                  !r90 = reading IRT score in 1990
                  !r92 = reading IRT score in 1992
```

Input For NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
        !s88 = science IRT score in 1988
        !s90 = science IRT score in 1990
        !s92 = science IRT score in 1992
        !h88 = history IRT score in 1988
        !h90 = history IRT score in 1990
        !h92 = history IRT score in 1992
        !female = scored 1 vs 0
        !stud_ses = student family ses in 1990 (f1ses)
        !per_adva = percent teachers with an MA or higher
        !private = private school (scored 1 vs 0)
        !catholic = catholic school (scored 1 vs 0)
        !private = 0, catholic = 0 implies public school
MISSING = BLANK;
CLUSTER = school;
USEV = r88 m88 s88 h88 r90 m90 s90 h90 r92 m92 s92 h92
female stud_ses per_adva private catholic mean_ses;
WITHIN = female stud_ses;
BETWEEN = per_adva private catholic mean_ses;
```

95

Input For NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
ANALYSIS:    TYPE = TWOLEVEL MISSING;
MODEL:       %WITHIN%
             fw1 BY r88-h88;
             fw2 BY r90-h90;
             fw3 BY r92-h92;
             r88 WITH r90; r90 WITH r92; r88 WITH r92;
             m88 WITH m90; m90 WITH m92; m88 WITH m92;
             s88 WITH s90; s90 WITH s92;
             h88 WITH h90; h90 WITH h92;
             fw1-fw3 ON female stud_ses;
             %BETWEEN%
             fb1 BY r88-h88;
             fb2 BY r90-h90;
             fb3 BY r92-h92;
             fb1-fb3 ON per_adva private catholic mean_ses;
OUTPUT:      SAMPSTAT STANDARDIZED TECH1 TECH8 MODINDICES;
```

96

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates

Summary Of Data

```
    Number of patterns       15
    Number of clusters      913

Average cluster size 15.572

    Estimated Intraclass Correlations for the Y Variables
```

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
R88	0.067	M88	0.129	S88	0.100
H88	0.105	R90	0.076	M90	0.117
S90	0.110	H90	0.106	R92	0.073
M92	0.111	S92	0.099	H92	0.091

97

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Tests Of Model Fit

```
Chi-Square Test of Model Fit
              Value                                   4883.539*
              Degrees of Freedom                           146
              P-Value                                   0.0000
              Scaling Correction Factor                  1.046
                 for MLR

Chi-Square Test of Model Fit for the Baseline Model
              Value                                 150256.855
              Degrees of Freedom                           202
              P-Value                                   0.0000

CFI/TLI
              CFI                                        0.968
              TLI                                        0.956

Loglikelihood
              H0 Value                             -487323.777
              H1 Value                             -484770.257
```

98

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
Information Criteria

          Number of Free Parameters                       94
          Akaike (AIC)                            974835.554
          Bayesian (BIC)                          975546.400
          Sample-Size Adjusted BIC                975247.676
            (n* = (n + 2) / 24)

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                                     0.048
SRMR (Standardized Root Mean Square Residual
          Value for Between                            0.041
          Value for Within                             0.027
```


Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Within Level					
FW1 BY					
R88	1.000	0.000	0.000	6.528	0.812
M88	0.940	0.010	94.856	6.135	0.804
S88	1.005	0.010	95.778	6.559	0.837
H88	1.041	0.011	97.888	6.796	0.837
FW2 BY					
R90	1.000	0.000	0.000	8.038	0.842
M90	0.911	0.008	109.676	7.321	0.838
S90	1.003	0.010	99.042	8.065	0.859
H90	0.939	0.008	113.603	7.544	0.855

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
 FW3        BY
   R92                1.000      0.000      0.000      8.460      0.832
   M92                0.939      0.009    101.473      7.946      0.845
   S92                1.003      0.011     90.276      8.482      0.861
   H92                0.934      0.009    102.825      7.905      0.858
 FW1        ON
   FEMALE            -0.403      0.128     -3.150     -0.062     -0.031
   STUD_SES           3.378      0.096     35.264      0.517      0.418
FW2         ON
   FEMALE            -0.621      0.157     -3.945     -0.077     -0.039
   STUD_SES           4.169      0.110     37.746      0.519      0.420
FW3         ON
   FEMALE            -1.027      0.169     -6.087     -0.121     -0.064
   STUD_SES           4.418      0.122     36.124      0.522      0.422
```

101

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
Residual Variances
   R88               22.021      0.383     57.464     22.021      0.341
   M88               20.618      0.338     61.009     20.618      0.354
   S88               18.383      0.323     56.939     18.383      0.299
   H88               19.805      0.370     53.587     19.805      0.300
   R90               26.546      0.491     54.033     26.546      0.291
   M90               22.756      0.375     60.748     22.756      0.298
   S90               23.150      0.383     60.516     23.150      0.262
   H90               21.002      0.403     52.124     21.002      0.270
   R92               31.821      0.617     51.562     31.821      0.308
   M92               25.213      0.485     52.018     25.213      0.285
   S92               25.155      0.524     47.974     25.155      0.259
   H92               22.479      0.489     46.016     22.479      0.265
   FW1               35.081      0.699     50.201      0.823      0.823
   FW2               53.079      1.005     52.806      0.822      0.822
   FW3               58.438      1.242     47.041      0.817      0.817
```

102

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
Between Level
FB1          BY
   R88              1.000      0.000      0.000      1.952      0.933
   M88              1.553      0.070     22.138      3.031      0.979
   S88              1.061      0.058     18.255      2.071      0.887
   H88              1.065      0.053     19.988      2.078      0.814
FB2          BY
   R90              1.000      0.000      0.000      2.413      0.923
   M90              1.407      0.058     24.407      3.395      1.003
   S90              1.220      0.062     19.697      2.943      0.946
   H90              0.973      0.047     20.496      2.348      0.829
FB3          BY
   R92              1.000      0.000      0.000      2.472      0.947
   M92              1.435      0.065     22.095      3.546      0.997
   S92              1.160      0.065     17.889      2.868      0.938
   H92              0.963      0.041     23.244      2.380      0.871
```

103

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
Between Level
FB1          ON
   PER_ADVA         0.217      0.292      0.742      0.111      0.024
   PRIVATE          0.303      0.344      0.883      0.155      0.042
   CATHOLIC        -0.696      0.277     -2.512     -0.357     -0.088
   MEAN_SES         2.513      0.206     12.185      1.288      0.672
FB2          ON
   PER_ADVA         0.280      0.338      0.828      0.116      0.025
   PRIVATE          0.453      0.392      1.155      0.188      0.051
   CATHOLIC        -0.538      0.334     -1.609     -0.223     -0.055
   MEAN_SES         3.054      0.239     12.805      1.266      0.660
FB3          ON
   PER_ADVA         0.473      0.375      1.261      0.192      0.041
   PRIVATE          0.673      0.435      1.547      0.272      0.074
   CATHOLIC        -0.206      0.372     -0.554     -0.084     -0.021
   MEAN_SES         3.142      0.258     12.169      1.271      0.663
```

104

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Residual Variances

R88	0.564	0.104	5.437	0.564	0.129
M88	0.399	0.093	4.292	0.399	0.042
S88	1.160	0.126	9.170	1.160	0.213
H88	2.203	0.203	10.839	2.203	0.338
R90	1.017	0.160	6.352	1.017	0.149
M90	-0.068	0.055	-1.225	-0.068	-0.006
S90	1.025	0.172	5.945	1.025	0.106
H90	2.518	0.216	11.636	2.518	0.313
R92	0.706	0.182	3.886	0.706	0.104
M92	0.076	0.076	1.000	0.076	0.006
S92	1.120	0.190	5.901	1.120	0.120
H92	1.810	0.211	8.599	1.810	0.242
FB1	1.979	0.245	8.066	0.520	0.520
FB2	3.061	0.345	8.875	0.526	0.526
FB3	3.010	0.409	7.363	0.493	0.493

105

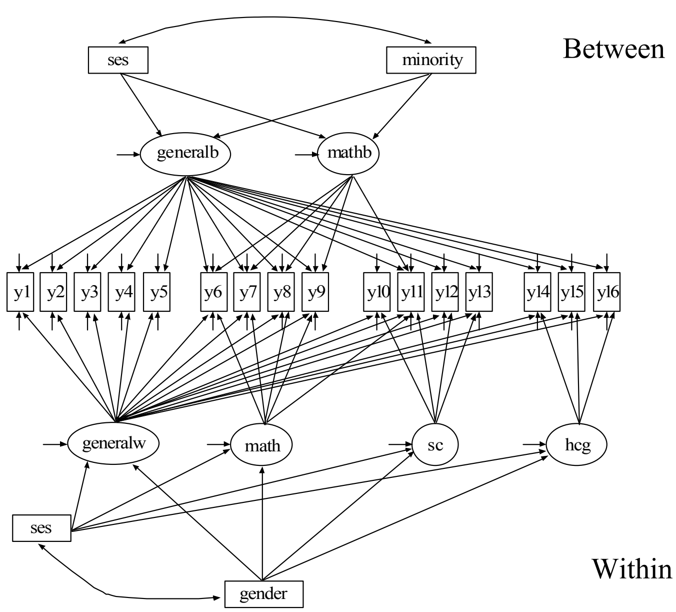
Multiple-Group, Two-Level Factor Analysis With Covariates

106

NELS Data

- The data—National Education Longitudinal Study (NELS:88)
  - Base year Grade 8—followed up in Grades 10 and 12
  - Students sampled within 1,035 schools—approximately 26 students per school
  - Variables—reading, math, science, history-citizenship-geography, and background variables
- Data for the analysis—reading, math, science, history-citizenship-geography, gender, individual SES, school SES, and minority status, n = 14,217 with 913 schools (clusters)

107



108

Input For NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools

```
TITLE:     NELS:88 with listwise deletion
           disaggregated model for two groups, public and
           catholic schools
DATA:      FILE IS EX831.DAT;;
VARIABLE:  NAMES = ses y1-y16 gender cluster minority group;
           CLUSTER = cluster;
           WITHIN = gender;
           BETWEEN = minority;
           GROUPING = group(1=public 2=catholic);
DEFINE:    minority = minority/5;
ANALYSIS:  TYPE = TWOLEVEL;
           H1ITER = 2500;
           MITER = 1000;
```

109

Input For NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

```
MODEL:     %WITHIN%
                 generalw BY y1* y2-y6 y8-y16 y7@1;
                 mathw BY y6* y8* y9* y11 y7@1;
                 scw BY y10 y11*.5 y12*.3 y13*.2;
                 hcgw BY y14*.7 y16*2 y15@1;
                 generalw WITH mathw-hcgw@0;
                 mathw WITH scw-hcgw@0;
                 scw WITH hcgw@0;
                 generalw mathw scw hcgw ON gender ses;
           %BETWEEN%
                 generalb BY y1* y2-y6 y8-y16 y7@1;
                 mathb BY y6* y8 y9 y11 y7@1;
                 y1-y16@0;
                 generalb WITH mathb@0;
                 generalb mathb ON ses minority;
```

110

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools

Summary Of Data

```
Group PUBLIC
   Number of clusters        195
     Size (s) Cluster ID with Size s
        1        68114   68519
        2        72872
        7        72765
        8        45991   72012
        9        68071
       10         7298   72187
       11        72463    7105   72405
       12        24083   68971    7737   68390
       13        45861   72219   72049
       14        68511   72148   72175   72176  25464
       15        68023   25071   68748   45928   7915   78324
       16        45362    7403   72415   77204  77219   72456
       17        45502   68487   45824    7203  24948    7829   72612    7892
                 25835    7591   68155   68295
```


Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

```
       18        72133   25580   24910   68614  25074   72990   68328   25404
                  7348
       19         7671   68662   68671   45385   7438    7332   25615   72799
                 68340   72956   25642   25658  24856   78283   68030
       20        72617   72715    7211   25422   7330   72292   72060   72993
                  7451   68461   78162   78232  72170   25130
       21        45394    7193   68180   24589   7205   25894   25958   68391
                 77254   77634   68448   45271   7584   25227   78598
       22        68254   68397   68648   72768   7192    7117    7119   68753
                 24813
       23        68456   25361    7157   25702  25804   45620   24858    7658
                 25163   45041   77351   45183  77684   78101   68788   68817
                  7792   78311   68048   68453
       24        77222   24053    7000   77403  24138   68297   78011   25536
                  7778   72042   25360   25977  45747    7616   78886
       25        68906   68720   25354   68427  72833   77268    7269   68520
                 77537   72075
       26        72973   45555   24828   68315  45087   25328   77710   25848
       27        45831   25618   68652   72080  45900   25208   45452    7103
```

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

28	25666	68809	25076	25224	68551
30	7343	45978	25722	45924	
31	77109	7230	68855		
32	25178				
33	45330	25745	25825		
35	25667				
36	72129				
37	25834				
38	45287				
39	45197	7090			
43	45366				

113

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

```
Group PUBLIC
  Number of clusters          195
  Average cluster size 21.292
  Estimated Intraclass Correlations for the Y Variables
```

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
Y1	.111	Y7	.100	Y12	.115
Y2	.105	Y8	.124	Y13	.185
Y3	.213	Y9	.069	Y14	.094
Y4	.160	Y10	.147	Y15	.132
Y5	.081	Y11	.105	Y16	.159
Y6	.159				

114

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

```
Group CATHOLIC

  Number of clusters          40
  Average cluster size 26.016

  Estimated Intraclass Correlations for the Y Variables

              Intraclass              Intraclass             Intraclass
   Variable  Correlation  Variable   Correlation  Variable  Correlation

   Y1            .010      Y7           .029       Y12          .056
   Y2            .039      Y8           .061       Y13          .176
   Y3            .180      Y9           .056       Y14          .078
   Y4            .091      Y10          .079       Y15          .071
   Y5            .055      Y11          .056       Y16          .154
   Y6            .118
```

115

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Tests Of Model Fit

```
Loglikelihood
              Value                              1716.922*
              Degrees of Freedom                      575
              P-Value                              0.0000
              Scaling Correction Factor             0.872
                 for MLR

Chi-Square Test of Model
              Value                             35476.471
              Degrees of Freedom                      608
              P-Value                              0.0000

CFI/TLI
              CFI                                   0.967
              TLI                                   0.965

Loglikelihood
              H0 Value                        -130332.921
              H1 Value                        -129584.053
```

116

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Group Public
Within Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
GENERALW ON					
GENDER	-0.193	0.029	-6.559	-0.256	-0.128
SES	0.233	0.016	14.269	0.309	0.279
MATHW ON					
GENDER	0.266	0.025	10.534	0.510	0.255
SES	0.054	0.014	3.879	0.103	0.093
SCW ON					
GENDER	0.452	0.032	14.005	0.961	0.480
SES	0.018	0.015	1.244	0.039	0.035
HCGW ON					
GENDER	0.152	0.023	6.588	0.681	0.341
SES	0.002	0.007	0.239	0.007	0.007

117

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Group Catholic
Within Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
GENERALW ON					
GENDER	-0.294	0.059	-5.000	-0.403	-0.201
SES	0.169	0.021	7.892	0.232	0.193
MATHW ON					
GENDER	0.332	0.051	6.478	0.627	0.313
SES	-0.030	0.017	-1.707	-0.056	-0.047
SCW ON					
GENDER	0.555	0.063	8.860	1.226	0.613
SES	-0.022	0.014	-1.592	-0.049	-0.041
HCGW ON					
GENDER	0.160	0.029	5.610	0.785	0.392
SES	0.001	0.007	0.089	0.003	0.002

118

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Group Public
Between Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
GENERALB ON					
SES	0.505	0.079	6.390	1.244	0.726
MINORITY	-0.217	0.088	-2.452	-0.534	-0.188
MATHB ON					
SES	0.198	0.070	2.825	0.984	0.574
MINORITY	-0.031	0.087	-0.354	-0.153	-0.054
GENERALB WITH					
MATHB	0.000	0.000	0.000	0.000	0.000
Intercepts					
GENERALB	0.000	0.000	0.000	0.000	0.000
MATHB	0.000	0.000	0.000	0.000	0.000

119

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Group Catholic
Between Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
GENERALB ON					
SES	0.262	0.067	3.929	0.975	0.538
MINORITY	-0.327	0.069	-4.707	-0.216	-0.573
MATHB ON					
SES	0.205	0.071	2.901	0.746	0.412
MINORITY	-0.213	0.095	-2.241	-0.778	-0.367
GENERALB WITH					
MATHB	0.000	0.000	0.000	0.000	0.000
Intercepts					
GENERALB	0.466	0.163	2.854	1.734	1.734
MATHB	0.573	0.177	3.239	2.087	2.087

120

Further Readings On Two-Level Factor Analysis

Harnqvist, K., Gustafsson, J.E., Muthén, B, & Nelson, G. (1994). Hierarchical models of ability at class and individual levels. Intelligence, 18, 165-187. (#53)

Hox, J. (2002). Multilevel analysis. Techniques and applications. Mahwah, NJ: Lawrence Erlbaum

Longford, N. T., & Muthén, B. (1992). Factor analysis for clustered observations. Psychometrika, 57, 581-597. (#41)

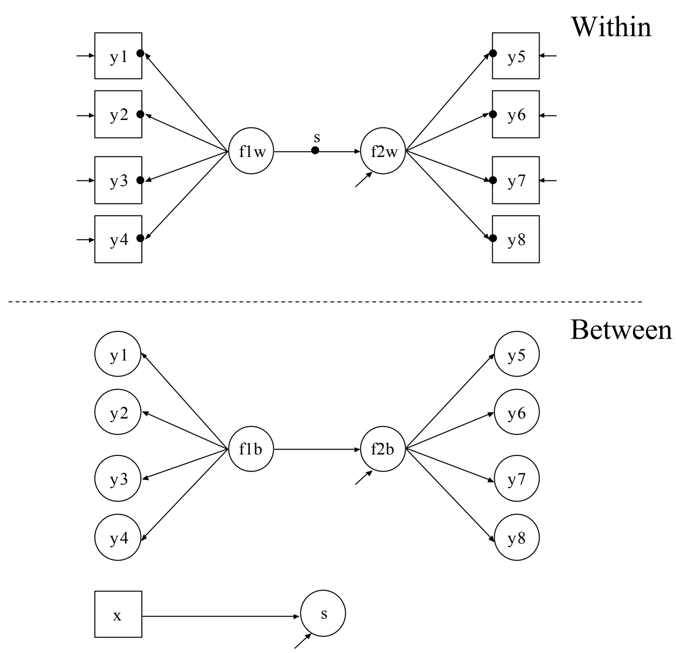
Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Muthén, B. (1990). Mean and covariance structure analysis of hierarchical data. Paper presented at the Psychometric Society meeting in Princeton, NJ, June 1990. UCLA Statistics Series 62. (#32)

Muthén, B. (1991). Multilevel factor analysis of class and student achievement components. Journal of Educational Measurement, 28, 338-354. (#37)

Muthén, B. (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research, 22, 376-398. (#55)

Two-Level SEM: Random Slopes For Regressions Among Factors



123

Input For A Two-Level SEM With A Random Slope

```
TITLE:      a twolevel SEM with a random slope

DATA:       FILE = etaeta3.dat;

VARIABLE:   NAMES ARE y1-y8 x clus;
            CLUSTER = clus;
            BETWEEN = x;

ANALYSIS:   TYPE = TWOLEVEL RANDOM MISSING;
            ALGORITHM = INTEGRATION;
```

124

Input For A Two-Level SEM
With A Random Slope (Continued)

```
MODEL:      %WITHIN%
            f1w BY y1@1
            y2 (1)
            y3 (2)
            y4 (3);
            f2w BY y5@1
            y6 (4)
            y7 (5)
            y8 (6);
            s | f2w ON f1w;

            %BETWEEN%
            f1b BY y1@1
            y2 (1)
            y3 (2)
            y4 (3);
            f2b BY y5@1
            y6 (4)
            y7 (5)
            y8 (6);
            f2b ON f1b;
            s ON x;
OUTPUT:     TECH1 TECH8;
```

125

Output Excerpts Two-Level SEM
With A Random Slope

Tests Of Model Fit

```
Loglikelihood

             H0 Value                         -12689.557

Information Criteria

             Number of Free Parameters                30
             Akaike (AIC)                      25439.114
             Bayesian (BIC)                    25585.122
             Sample-Size Adjusted BIC          25489.843
                (n* = (n + 2) / 24)
```

126

Output Excerpts Two-Level SEM With A Random Slope (Continued)

Model Results

		Estimates	S.E.	Est./S.E.
Within Level				
F1W	BY			
Y1		1.000	0.000	0.000
Y2		0.992	0.035	28.597
Y3		0.978	0.041	23.593
Y4		1.001	0.037	26.884
F2W	BY			
Y5		1.000	0.000	0.000
Y6		0.978	0.028	34.417
Y7		1.049	0.030	35.174
Y8		1.008	0.026	38.090
F1W	WITH			
F2W		0.000	0.000	0.000

127

Output Excerpts Two-Level SEM With A Random Slope (Continued)

	Estimates	S.E.	Est./S.E.
Variances			
F1W	1.016	0.082	12.325
F2W	0.580	0.063	9.144
Residual Variances			
Y1	0.979	0.063	15.517
Y2	0.949	0.056	16.854
Y3	1.052	0.060	17.406
Y4	0.971	0.053	18.174
Y5	1.039	0.057	18.187
Y6	1.062	0.058	18.292
Y7	0.941	0.058	16.191
Y8	1.076	0.060	17.835

128

Output Excerpts Two-Level SEM With A Random Slope (Continued)

	Estimates	S.E.	Est./S.E.
Between Level			
F1B BY			
Y1	1.000	0.000	0.000
Y2	0.992	0.035	28.597
Y3	0.978	0.041	23.593
Y4	1.001	0.037	26.884
F2B BY			
Y5	1.000	0.000	0.000
Y6	0.978	0.028	34.417
Y7	1.049	0.030	35.174
Y8	1.008	0.026	38.090
F2B ON			
F1B	0.180	0.080	2.248

129

Output Excerpts Two-Level SEM With A Random Slope (Continued)

	Estimates	S.E.	Est./S.E.
S ON			
X	0.999	0.082	12.150
Intercepts			
Y1	-0.099	0.063	-1.560
Y2	-0.011	0.064	-0.175
Y3	-0.069	0.067	-1.034
Y4	-0.001	0.065	-0.017
Y5	0.030	0.062	0.475
Y6	-0.008	0.064	-0.129
Y7	0.041	0.064	0.635
Y8	0.002	0.071	0.035
S	0.777	0.073	10.604
Variances			
F1B	0.568	0.096	5.900
Residual Variances			
F2B	0.237	0.056	4.211
S	0.420	0.088	4.756

130

Multilevel Estimation, Testing, Modification, And Identification

Estimators

- Muthén's limited information estimator (MUML) – random intercepts
  - ESTIMATOR = MUML
  - Muthén's limited information estimator for unbalanced data
  - Maximum likelihood for balanced data
- Full-information maximum likelihood (FIML) – random intercepts and random slopes
  - ESTIMATOR = ML, **MLR**, MLF
  - Full-information maximum likelihood for balanced and unbalanced data
  - Robust maximum likelihood estimator
  - MAR missing data
  - Asparouhov and Muthén

131

Multilevel Estimation, Testing, Modification, And Identification (Continued)

Tests of Model Fit

- MUML – chi-square, robust chi-square, CFI, TLI, RMSEA, and SRMR
- FIML – chi-square, robust chi-square, CFI, TLI, RMSEA, and SRMR
- FIML with random slopes – no tests of model fit

Model Modification

- MUML – modification indices not available
- FIML – modification indices available

Model identification is the same as for CFA for both the between and within parts of the model.

132

Practical Issues Related To The Analysis Of Multilevel Data

Size Of The Intraclass Correlation

- Small intraclass correlations can be ignored but important information about between-level variability may be missed by conventional analysis
- The importance of the size of an intraclass correlation depends on the size of the clusters
- Intraclass correlations are attenuated by individual-level measurement error
- Effects of clustering not always seen in intraclass correlations

133

Practical Issues Related To The Analysis Of Multilevel Data (Continued)

Within-Level And Between-Level Variables

- Variables measured on the individual level can be used in both the between and within parts of the model or only in the within part of the model (WITHIN=)
- Variables measured on the between level can be used only in the between part of the model (BETWEEN=)

Sample Size

- There should be at least 30-50 between-level units (clusters)
- Clusters with only one observation are allowed

134

Steps In SEM Multilevel Analysis For Continuous Outcomes

1) Explore SEM model using the sample covariance matrix from the total sample
2) Estimate the SEM model using the pooled-within sample covariance matrix with sample size n - G
3) Investigate the size of the intraclass correlations and DEFF's
4) Explore the between structure using the estimated between covariance matrix with sample size G
5) Estimate and modify the two-level model suggested by the previous steps

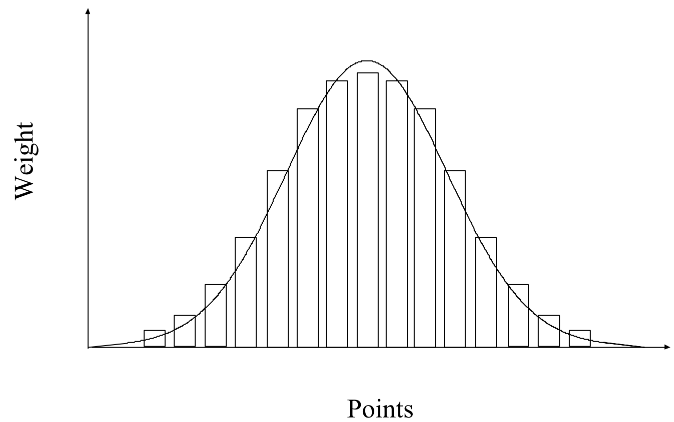
Muthén, B. (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research, 22, 376-398. (#55)

135

Technical Aspects Of Multilevel Modeling

136

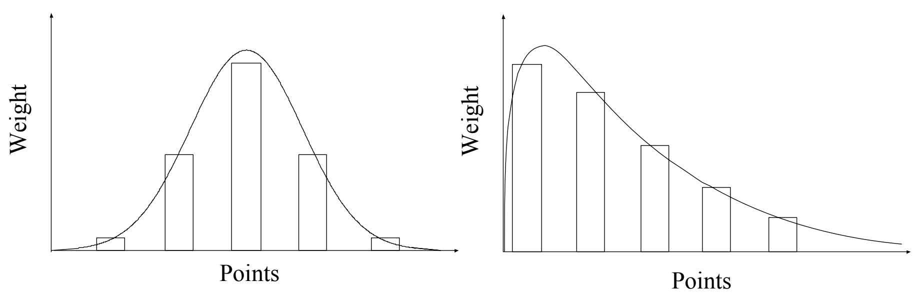
Numerical Integration With A Normal Latent Variable Distribution



Fixed weights and points

137

Non-Parametric Estimation Of The Random Effect Distribution



Estimated weights and points
(class probabilities and class means)

138

Numerical Integration

Numerical integration is needed with maximum likelihood estimation when the posterior distribution for the latent variables does not have a closed form expression. This occurs for models with categorical outcomes that are influenced by continuous latent variables, for models with interactions involving continuous latent variables, and for certain models with random slopes such as multilevel mixture models.

When the posterior distribution does not have a closed form, it is necessary to integrate over the density of the latent variables multiplied by the conditional distribution of the outcomes given the latent variables. Numerical integration approximates this integration by using a weighted sum over a set of integration points (quadrature nodes) representing values of the latent variable.

Numerical Integration (Continued)

Numerical integration is computationally heavy and thereby time-consuming because the integration must be done at each iteration, both when computing the function value and when computing the derivative values. The computational burden increases as a function of the number of integration points, increases linearly as a function of the number of observations, and increases exponentially as a function of the dimension of integration, that is, the number of latent variables for which numerical integration is needed.

Practical Aspects Of Numerical Integration

- Types of numerical integration available in Mplus with or without adaptive quadrature
  - Standard (rectangular, trapezoid) – default with 15 integration points per dimension
  - Gauss-Hermite
  - Monte Carlo
- Computational burden for latent variables that need numerical integration

• One or two latent variables	Light
• Three to five latent variables	Heavy
• Over five latent variables	Very heavy

141

Practical Aspects Of Numerical Integration (Continued)

- Suggestions for using numerical integration
  - Start with a model with a small number of random effects and add more one at a time
  - Start with an analysis with TECH8 and MITERATIONS=1 to obtain information from the screen printing on the dimensions of integration and the time required for one iteration and with TECH1 to check model specifications
  - With more than 3 dimensions, reduce the number of integration points to 5 or 10 or use Monte Carlo integration with the default of 500 integration points
  - If the TECH8 output shows large negative values in the column labeled ABS CHANGE, increase the number of integration points to improve the precision of the numerical integration and resolve convergence problems

142

Technical Aspects Of Numerical Integration

Maximum likelihood estimation using the EM algorithm computes in each iteration the posterior distribution for normally distributed latent variables f,

$$[f \mid y] = [f]\,[y \mid f] \,/\, [y], \qquad (97)$$

where the marginal density for $[y]$ is expressed by integration

$$[y] = \int [f]\,[y \mid f]\,df. \qquad (98)$$

- Numerical integration is not needed for normally distributed y - the posterior distribution is normal

143

Technical Aspects Of Numerical Integration (Continued)

- Numerical integration needed for:
  - Categorical outcomes u influenced by continuous latent variables f, because $[u]$ has no closed form
  - Latent variable interactions f x x, f x y, f_1 x f_2, where $[y]$ has no closed form, for example

$$[y] = \int [f_1, f_2]\,[y \mid f_1, f_2, f_1 f_2]\,df_1\,df_2 \qquad (99)$$

  - Random slopes, e.g. with two-level mixture modeling

Numerical integration approximates the integral by a sum

$$[y] = \int [f]\,[y \mid f]\,df = \sum_{k=1}^{K} w_k\,[y \mid f_k] \qquad (100)$$

144

Multivariate Approach To Multilevel Modeling

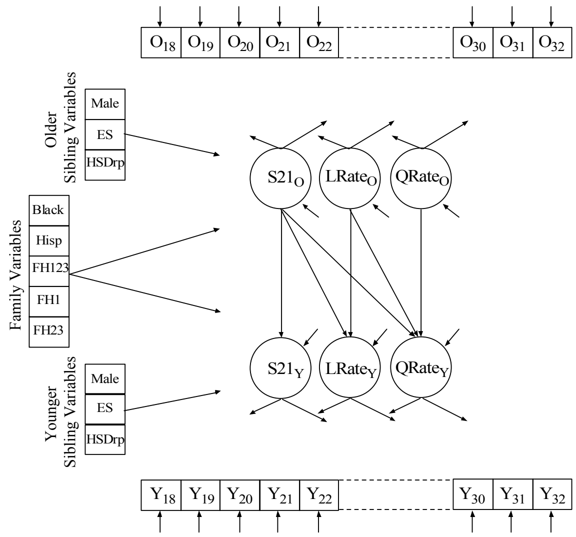
145

Multivariate Modeling Of Family Members

- Multilevel modeling: clusters independent, model for between- and within-cluster variation, units within a cluster statistically equivalent
- Multivariate approach: clusters independent, model for all variables for each cluster unit, different parameters for different cluster units.
  - Used in latent variable growth modeling where the cluster units are the repeated measures over time
  - Allows for different cluster sizes by missing data techniques
  - More flexible than the multilevel approach, but computationally convenient only for applications with small cluster sizes (e.g. twins, spouses)

146

Figure 1. A Longitudinal Growth Model of Heavy Drinking for Two-Sibling Families



Source: Khoo, S.T. & Muthen, B. (2000). Longitudinal data on families: Growth modeling alternatives. Multivariate Applications in Substance Use Research, J. Rose, L. Chassin, C. Presson & J. Sherman (eds.), Hillsdale, N.J.: Erlbaum, pp. 43-78.

147

Three-Level Modeling As Single-Level Analysis

Doubly multivariate:

- Repeated measures in wide, multivariate form
- Siblings in wide, multivariate form

It is possible to do four-level by TYPE = TWOLEVEL, for instance families within geographical segments

148

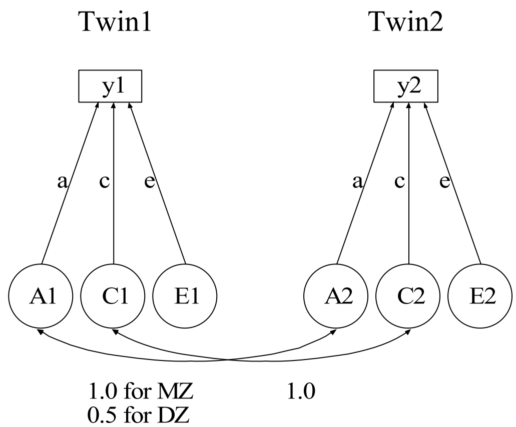
Input For Multivariate Modeling Of Family Data

```
TITLE:      Multivariate modeling of family data
            one observation per family
DATA:       FILE IS multi.dat;
VARIABLE:   NAMES ARE o18-o32 y18-y32 omale oes ohsdrop ymale yoes
            yhsdrop black hisp fh123 fh1 fh23;
MODEL:      s21o lrateo qrateo | o18@0 o19@1 o20@2 o21@3 o22@4
            o23@5 o24@6 o25@7 o26@8 o27@9 o28@10 o29@11 o30@12
            o31@13 o32@14;
            s21y lratey qratey | y18@0 y19@1 y20@2 y21@3 y22@4 y23@5
            y24@6 y25@7 y26@8 y27@9 y28@10 y29@11 y30@12
            y31@13 y32@14;
            s12o ON omale oes ohsdrop black hisp fh123 fh1 fh23;
            221y ON ymale yes yhsdrop black hisp fh123 fh1 fh23;
            s21y ON s21o;
            lratey ON s21o lrateo;
            qratey ON s21o lrateo qrateo;
```

149

Twin Modeling

150



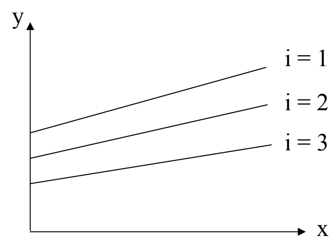
Neale & Cardon (1992)
Prescott (2004)

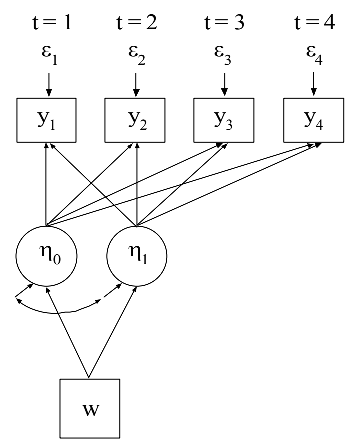
151

Multilevel Growth Models

152

Individual Development Over Time





(1) $y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}$

(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

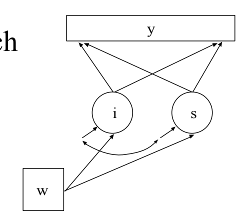
153

Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- Wide: Multivariate, Single-Level Approach

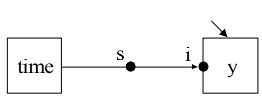
  $y_{ti} = i_i + s_i \times \text{time}_{ti} + \varepsilon_{ti}$

  i_i regressed on w_i
  s_i regressed on w_i

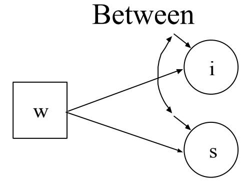


- Long: Univariate, 2-Level Approach (CLUSTER = id)

Within



Between



The intercept *i* is called *y* in Mplus

154

Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long (Continued)

- Wide (one person):

		t1	t2	t3	t1	t2	t3	
Person i:	id	y1	y2	y3	x1	x2	x3	w

- Long (one cluster):

Person i:	t1	id	y1	x1	w
	t2	id	y2	x2	w
	t3	id	y3	x3	w

155

Three-Level Modeling In Multilevel Terms

Time point *t*, individual *i*, cluster *j*.

y_{tij} : individual-level, outcome variable
a_{1tij} : individual-level, time-related variable (age, grade)
a_{2tij} : individual-level, time-varying covariate
x_{ij} : individual-level, time-invariant covariate
w_j : cluster-level covariate

Three-level analysis (Mplus considers Within and Between)

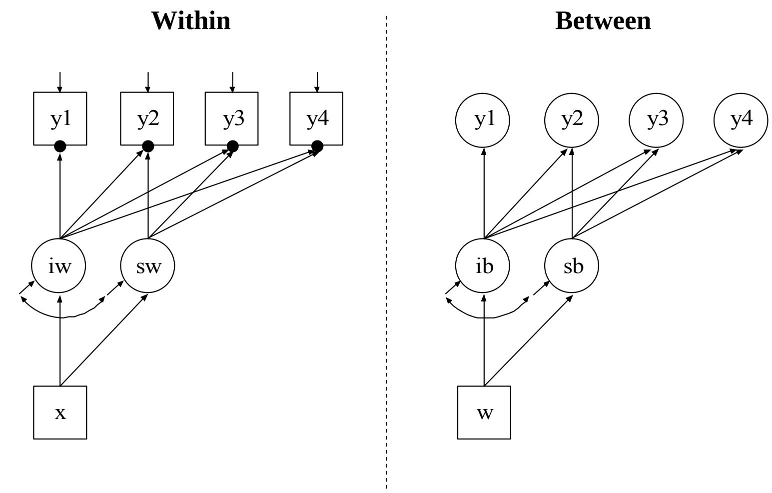
$$Level\ 1\ (Within)\ :\ y_{tij} = \pi_{0ij} + \pi_{1ij}\, a_{1tij} + \pi_{2tij}\, a_{2tij} + e_{tij}, \qquad (1)$$

$$Level\ 2\ (Within)\ : \begin{cases} \pi_{0ij} = \beta_{00j} + \beta_{01j}\, x_{ij} + r_{0ij}, & \rightarrow iw \\ \pi_{1ij} = \beta_{10j} + \beta_{11j}\, x_{ij} + r_{1ij}, \\ \pi_{2tij} = \beta_{20tj} + \beta_{21tj}\, x_{ij} + r_{2tij}. \end{cases} \qquad (2)$$

$$Level\ 3\ (Between) : \begin{cases} \beta_{00j} = \gamma_{000} + \gamma_{001}\, w_j + u_{00j}, & \rightarrow ib \\ \beta_{10j} = \gamma_{100} + \gamma_{101}\, w_j + u_{10j}, \\ \beta_{20tj} = \gamma_{200t} + \gamma_{201t}\, w_j + u_{20tj}, \\ \beta_{01j} = \gamma_{010} + \gamma_{011}\, w_j + u_{01j}, \\ \beta_{11j} = \gamma_{110} + \gamma_{111}\, w_j + u_{11j}, \\ \beta_{21tj} = \gamma_{2t0} + \gamma_{2t1}\, w_j + u_{2tj}. \end{cases} \qquad (3)$$

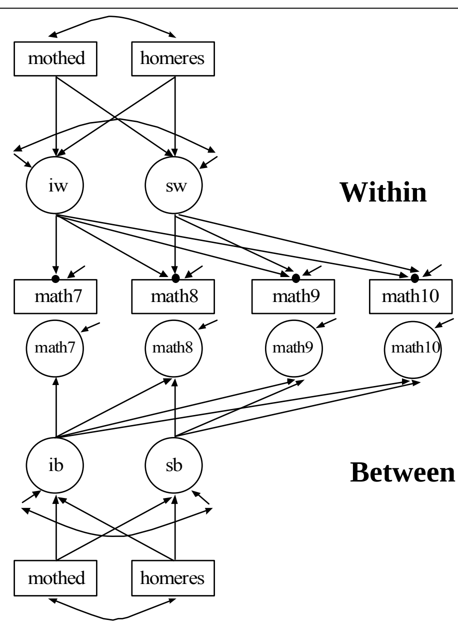
156

Two-Level Growth Modeling (Three-Level Modeling)



157

LSAY Two-Level Growth Model



158

Input For LSAY Two-Level Growth Model With Free Time Scores And Covariates

```
TITLE:     LSAY two-level growth model with free time scores
           and covariates
DATA:      FILE IS lsay98.dat;
           FORMAT IS 3f8 f8.4 8f8.2 3f8 2f8.2;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10 mothed homeres;
           CLUSTER = school;
ANALYSIS:  TYPE = TWOLEVEL;
           ESTIMATOR = MUML;
```

159

Input For LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

```
MODEL:     %WITHIN%
           iw sw | math7@0 math8@1
           math9*2  (1)
           math10*3 (2);
           iw sw ON mothed homeres;
           %BETWEEN%
           ib sb | math7@0 math8@1
           math9*2  (1)
           math10*3 (2);
           ib sb ON mothed homeres;

OUTPUT     SAMPSTAT STANDARDIZED RESIDUAL;
```

160

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates

Summary of Data

```
Number of clusters       50

  Size (s)  Cluster ID with Size s
    1         114
    2         136
    6         132     304

   34         104
   39         309
   40         302

Average cluster size 18.627

Estimated Intraclass Correlations for the Y Variables
```

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
MATH7	0.199	MATH8	0.149	MATH9	0.168
MATH10	0.165				

161

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Tests Of Model Fit

```
Chi-square Test of Model Fit
            Value                        24.058*
            Degrees of Freedom               14
            P-Value                      0.0451
CFI / TLI
            CFI                           0.997
            TLI                           0.995
RMSEA (Root Mean Square Error Of Approximation)
            Estimate                      0.028
SRMR (Standardized Root Mean Square Residual)
            Value for Between             0.048
            Value for Within              0.007
```

162

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Model Results

Within Level

```
SW       BY
   MATH8              1.000     0.000     0.000     1.073     0.128
   MATH9              2.487     0.163    15.220     2.670     0.288
   MATH10             3.589     0.223    16.076     3.853     0.368
IW       ON
   MOTHED             1.780     0.232     7.665     0.246     0.226
   HOMERES            0.892     0.221     4.031     0.124     0.173
SW       ON
   MOTHED             0.053     0.063     0.836     0.049     0.045
   HOMERES            0.135     0.044     3.047     0.125     0.176
```

163

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

```
SW       WITH
   IW                 2.112     0.522     4.044     0.273     0.273
HOMERES  WITH
   MOTHED             0.261     0.039     6.709     0.261     0.203
Residual Variances
   MATH7             12.748     1.434     8.888    12.748     0.197
   MATH8             12.298     0.893    13.771    12.298     0.174
   MATH9             14.237     1.132    12.578    14.237     0.166
   MATH10            24.829     2.230    11.133    24.829     0.226
   IW                47.060     3.069    15.333     0.903     0.903
   SW                 1.110     0.286     3.879     0.964     0.964
Variances
   MOTHED             0.841     0.049    17.217     0.841     1.000
   HOMERES            1.970     0.069    28.643     1.970     1.000
```

164

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

Between Level

	Estimates	S.E.	Est./S.E.	Std	StdYX
SB BY					
MATH8	1.000	0.000	0.000	0.196	0.052
MATH9	2.487	0.163	15.220	0.488	0.119
MATH10	3.589	0.223	16.076	0.704	0.115
IB ON					
MOTHED	**-1.225**	**2.587**	**-0.474**	**-0.362**	**-0.107**
HOMERES	**7.160**	**1.847**	**3.876**	**2.117**	**1.011**
SB ON					
MOTHED	**0.995**	**0.647**	**1.538**	**5.073**	**1.493**
HOMERES	**0.017**	**0.373**	**0.045**	**0.086**	**0.041**
SB WITH					
IB	0.382	0.248	1.538	0.575	0.575

165

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
HOMERES WITH					
MOTHED	0.103	0.019	5.488	0.103	0.733
Residual Variances					
MATH7	2.059	0.552	3.732	2.059	0.153
MATH8	0.544	0.268	2.033	0.544	0.039
MATH9	0.105	0.213	0.493	0.105	0.006
MATH10	1.395	0.504	2.767	1.395	0.067
IB	1.428	1.690	0.845	0.125	0.125
SB	-0.051	0.071	-0.713	-1.321	-1.321
Variances					
MOTHED	0.087	0.023	3.801	0.087	1.000
HOMERES	0.228	0.056	4.066	0.228	1.000
Means					
MOTHED	2.307	0.043	53.277	2.307	7.838
HOMERES	3.108	0.062	50.375	3.108	6.509
Intercepts					
IB	33.510	2.678	12.512	9.909	9.909
SB	0.163	0.776	0.210	0.830	0.830

166

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

R-Square

Within Level

Observed Variable	R-Square
MATH7	0.803
MATH8	0.826
MATH9	0.834
MATH10	0.774

Latent Variable	R-Square
IW	0.097
SW	0.036

167

Output Excerpts LSAY Two-Level Growth Model With Free Time Scores And Covariates (Continued)

R-Square

Between Level

Observed Variable	R-Square
MATH7	0.847
MATH8	0.961
MATH9	0.994
MATH10	0.933

Latent Variable	R-Square	
IW	0.875	
SW	Undefined	0.23207E+01

168

Further Readings On Three-Level Growth Modeling

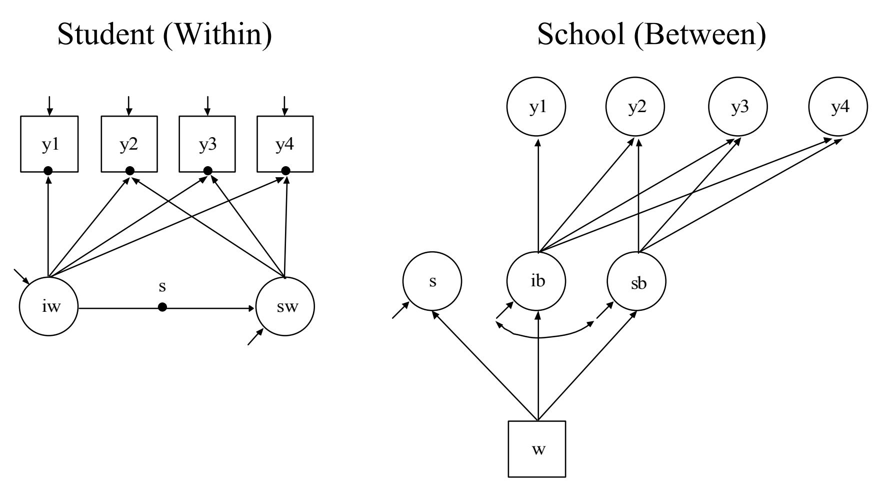
Muthén, B. (1997). Latent variable modeling with longitudinal and multilevel data. In A. Raftery (ed), Sociological Methodology (pp. 453-480). Boston: Blackwell Publishers. (#73)

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

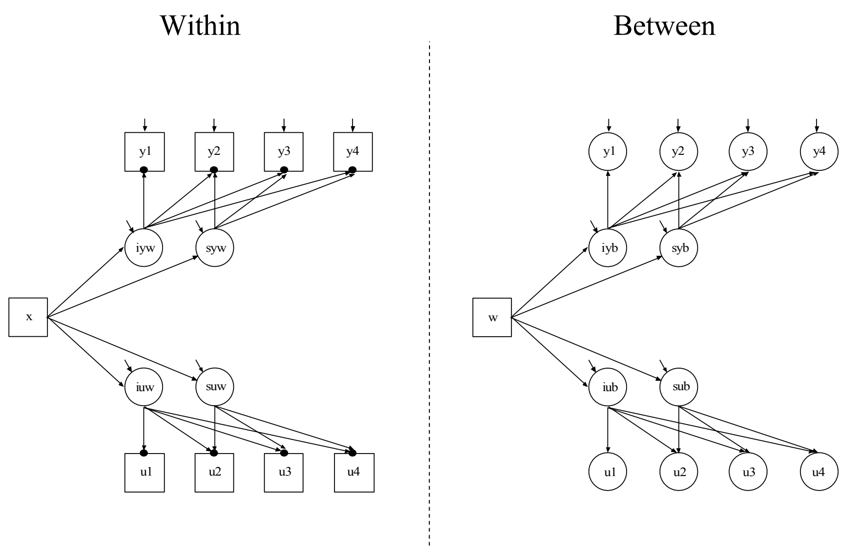
169

Multilevel Modeling With A Random Slope For Latent Variables



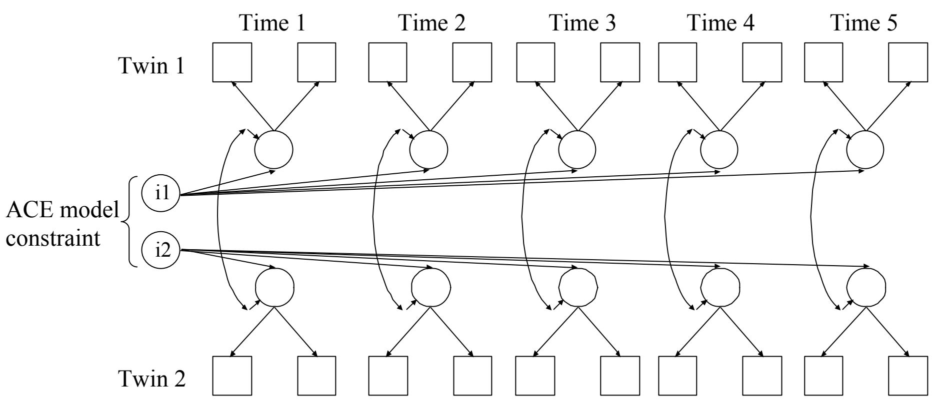
170

Two-Level, Two-Part Growth Modeling



Multiple Indicator Growth Modeling As Two-Level Analysis

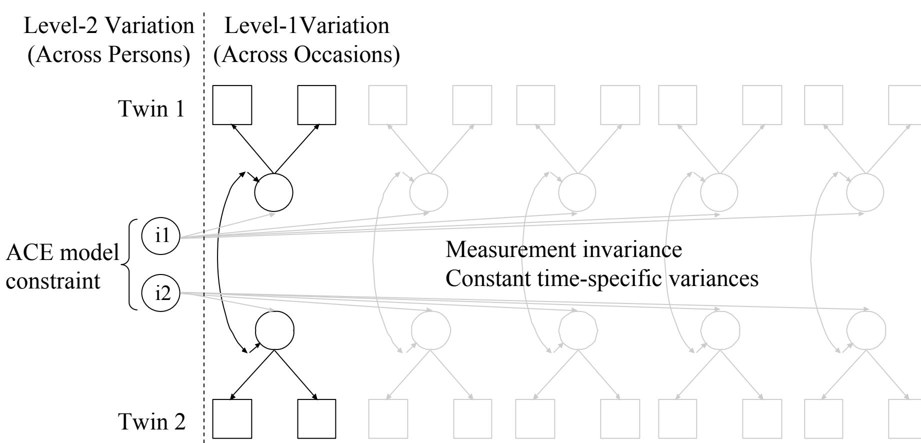
Wide Data Format, Single-Level Approach



20 variables, 12 factors, 10 dimensions of integration for ML
ML very hard, WLS easy

173

Long Format, Two-Level Approach



4 variables, 2 Level-2 and 2 Level-1 factors, 4 dimensions of integration for ML
ML feasible, WLS in development

174

Multilevel Discrete-Time Survival Analysis

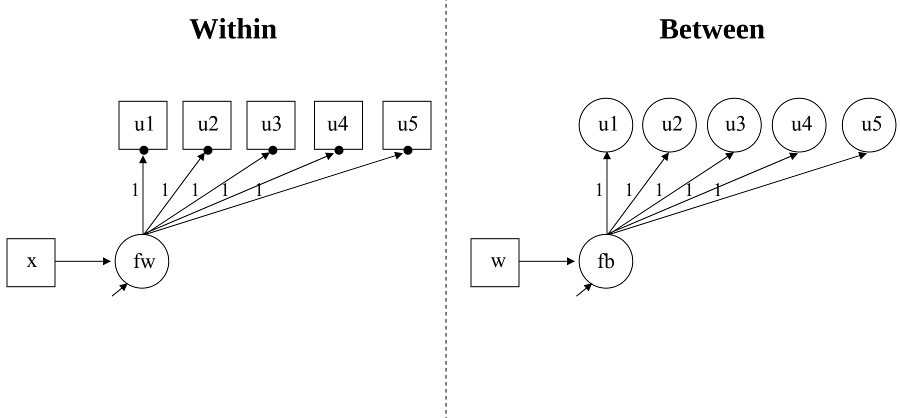
175

Multilevel Discrete-Time Survival Analysis

- Muthén and Masyn (2005) in Journal of Educational and Behavioral Statistics
- Masyn dissertation
- Asparouhov and Muthén

176

Multilevel Discrete-Time Survival Frailty Modeling



Vermunt (2003)

177

References

(To request a Muthén paper, please email bmuthen@ucla.edu.)

Cross-sectional Data

Asparouhov, T. (2005). Sampling weights in latent variable modeling. Structural Equation Modeling, 12, 411-434.

Chambers, R.L. & Skinner, C.J. (2003). Analysis of survey data. Chichester: John Wiley & Sons.

Harnqvist, K., Gustafsson, J.E., Muthén, B. & Nelson, G. (1994). Hierarchical models of ability at class and individual levels. Intelligence, 18, 165-187. (#53)

Heck, R.H. (2001). Multilevel modeling with SEM. In G.A. Marcoulides & R.E. Schumacker (eds.), New developments and techniques in structural equation modeling (pp. 89-127). Lawrence Erlbaum Associates.

Hox, J. (2002). Multilevel analysis. Techniques and applications. Mahwah, NJ: Lawrence Erlbaum.

Kaplan, D. & Elliott, P.R. (1997). A didactic example of multilevel structural equation modeling applicable to the study of organizations. Structural Equation Modeling: A Multidisciplinary Journal, 4, 1-24.

Kaplan, D. & Ferguson, A.J (1999). On the utilization of sample weights in latent variable models. Structural Equation Modeling, 6, 305-321.

178

References (Continued)

Kaplan, D. & Kresiman, M.B. (2000). On the validation of indicators of mathematics education using TIMSS: An application of multilevel covariance structure modeling. International Journal of Educational Policy, Research, and Practice, 1, 217-242.

Korn, E.L. & Graubard, B.I (1999). Analysis of health surveys. New York: John Wiley & Sons.

Kreft, I. & de Leeuw, J. (1998). Introducing multilevel modeling. Thousand Oakes, CA: Sage Publications.

Larsen & Merlo (2005). Appropriate assessment of neighborhood effects on individual health: Integrating random and fixed effects in multilevel logistic regression. American Journal of Epidemiology, 161, 81-88.

Longford, N.T., & Muthén, B. (1992). Factor analysis for clustered observations. Psychometrika, 57, 581-597. (#41)

Ludtke Marsh, Robitzsch, Trautwein, Asparouhov, Muthen (2007). Analysis of group level effects using multilevel modeling: Probing a latent covariate approach. Submitted for publication.

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

179

References (Continued)

Muthén, B. (1990). Mean and covariance structure analysis of hierarchical data. Paper presented at the Psychometric Society meeting in Princeton, N.J., June 1990. UCLA Statistics Series 62. (#32)

Muthén, B. (1991). Multilevel factor analysis of class and student achievement components. Journal of Educational Measurement, 28, 338-354. (#37)

Muthén, B. (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research, 22, 376-398. (#55)

Muthén, B., Khoo, S.T. & Gustafsson, J.E. (1997). Multilevel latent variable modeling in multiple populations. (#74)

Muthén, B. & Satorra, A. (1995). Complex sample data in structural equation modeling. In P. Marsden (ed.), Sociological Methodology 1995, 216-316. (#59)

Neale, M.C. & Cardon, L.R. (1992). Methodology for genetic studies of twins and families. Dordrecth, The Netherlands: Kluwer.

Patterson, B.H., Dayton, C.M. & Graubard, B.I. (2002). Latent class analysis of complex sample survey data: application to dietary data. Journal of the American Statistical Association, 97, 721-741.

Prescott, C.A. (2004). Using the Mplus computer program to estimate models for continuous and categorical data from twins. Behavior Genetics, 34, 17- 40.

180

References (Continued)

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Skinner, C.J., Holt, D. & Smith, T.M.F. (1989). Analysis of complex surveys. West Sussex, England, Wiley.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

Stapleton, L. (2002). The incorporation of sample weights into multilevel structural equation models. Structural Equation Modeling, 9, 475-502.

Vermunt, J.K. (2003). Multilevel latent class models. In Stolzenberg, R.M. (Ed.), Sociological Methodology (pp. 213-239). New York: American Sociological Association.

Longitudinal Data

Choi, K.C. (2002). Latent variable regression in a three-level hierarchical modeling framework: A fully Bayesian approach. Doctoral dissertation, University of California, Los Angeles.

181

References (Continued)

Khoo, S.T. & Muthén, B. (2000). Longitudinal data on families: Growth modeling alternatives. Multivariate applications in substance use research, J. Rose, L. Chassin, C. Presson & J. Sherman (eds.), Hillsdale, N.J.: Erlbaum, pp. 43-78. (#79)

Masyn, K. E. (2003). Discrete-time survival mixture analysis for single and recurrent events using latent variables. Doctoral dissertation, University of California, Los Angeles.

Muthén, B. (1997). Latent variable modeling with longitudinal and multilevel data. In A. Raftery (ed.) Sociological Methodology (pp. 453-480). Boston: Blackwell Publishers.

Muthén, B. (1997). Latent variable growth modeling with multilevel data. In M. Berkane (ed.), Latent variable modeling with application to causality (149-161), New York: Springer Verlag.

Muthén, B. & Masyn, K. (in press). Discrete-time survival mixture analysis. Journal of Educational and Behavioral Statistics, Spring 2005.

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications. Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

182

References (Continued)

Seltzer, M., Choi, K., Thum, Y.M. (2002). Examining relationships between where students start and how rapidly they progress: Implications for conducting analyses that help illuminate the distribution of achievement within schools. CSE Technical Report 560. CRESST, University of California, Los Angeles.

General

Mplus Analysis

Asparouhov, T. & Muthén, B. (2003a). Full-information maximum-likelihood estimation of general two-level latent variable models. In preparation.

Asparouhov, T. & Muthén, B. (2003b). Maximum-likelihood estimation in general latent variable modeling. In preparation.

Muthén, B. (2002). Beyond SEM: General latent variable modeling. Behaviormetrika, 29, 81-117.

Muthén, B. & Asparouhov, T. (2003b). Advances in latent variable modeling, part II: Integrating continuous and categorical latent variable modeling using Mplus. In preparation.

183

References (Continued)

Numerical Integration

Aitkin, M. A general maximum likelihood analysis of variance components in generalized linear models. Biometrics, 1999, 55, 117-128.

Bock, R.D. & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: Application of an EM algorithm. Psychometrika, 46, 443-459.

184